Name Answer Key

Statistics; Sec 5.2a – Some Probability Rules (Multiplication)

Answer all questions using complete sentences.

1. If two events are mutually exclusive, can they occur concurrently? Explain.

   No. by definition mutually exclusive events cannot occur together.

2. Given $P(A) = 0.2$ and $P(B) = 0.4$:

   a. If A and B, are independent events, compute $P(A\ and\ B)$.

   $$P(A \text{ and } B) = P(A) \cdot P(B)$$
   $$= 0.2 \times 0.4$$
   $$= 0.08$$

   b. If $P(A|B) = 0.1$, compute $P(A\ and\ B)$.

   $$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$
   $$0.1 = \frac{P(A \text{ and } B)}{0.4}$$

   $P(A \text{ and } B) = 0.04$

3. Given $P(A) = 0.2, P(B) = 0.5, P(A|B) = 0.3$, compute $P(A\ and\ B)$.

   $$P(A \text{ and } B) = P(B) \cdot P(A|B)$$
   $$= 0.5 \times 0.3$$
   $$= 0.15$$

4. Given $P(A^c) = 0.8, P(B) = 0.3, P(B|A) = 0.2$, compute $P(A\ and\ B)$.

   $$P(A \text{ and } B) = P(A) \cdot P(B|A)$$
   $$= (1 - .8) \cdot 0.2$$
   $$= 0.04$$

5. Lisa is making up questions for a small quiz on probability. She assigns these probabilities: $P(A) = 0.3, P(B) = 0.3, P(A\ and\ B) = 0.4$. What is wrong with these probability assignments?

   P(A and B) cannot be greater than P(A) or P(B).

6. Consider the following events for a driver selected at random from the general population:

$A =$ driver is under 25 years old

$B =$ driver has received a speeding ticket

Translate each of the following phrases into symbols.

a. The probability the driver has received a speeding ticket and is under 25 years old.

P(A and B)

b. The probability that a driver who is under 25 years old has received a speeding ticket.

$P(B|A)$

c. The probability a driver who has received a speeding ticket is 25 years old or older.

$P(A^c|B)$

d. The probability the driver is under 25 years old or has received a speeding ticket.

P(A or B)

e. The probability the driver has not received a speeding ticket or is under 25 years old.

P(B^c or A)

7. You roll two fair dice, a green one and a red one.

a. Are the outcomes on the dice independent?

Yes.

b. Find $P(5 \text{ on green die } \textit{and } 3 \text{ on red die})$.

$P(5) \cdot P(3) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \approx 0.028$

c. Find $P(3 \text{ on green die } \textit{and } 5 \text{ on red die})$.

$P(3) \cdot P(5) = \frac{1}{36} \approx 0.028$

d. Find $P\big((1 \text{ on green die } \textit{and } 3 \text{ on red die}) \textit{ or } (2 \text{ on green die } \textit{and } 1 \text{ on red die})\big)$.

$\frac{1}{36} + \frac{1}{36} = \frac{1}{18} \approx 0.056$

Problems 8 – 9 involve a standard deck of 52 playing cards. In such a deck of cards there are four suits of 13 cards each. The four suits are: hearts, diamonds, clubs, and spades. The 26 cards included in hearts and diamonds are red in color. The 26 cards included in clubs and spades are black in color. The 13 cards in each suit are: 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, and Ace. This means there are four Aces, four Kings, four Queens, four 10s, etc. down to four 2s in each deck.

8. You draw two cards from a standard deck of 52 cards without replacing the first one before drawing the second.

   a. Are the outcomes on the two cards independent? Why?

      No, after the 1st draw the sample space and probabilities change.

   b. Find $P(\text{Ace on 1st card } \textit{and} \text{ King on 2nd})$.

      $P(\text{Ace}) \cdot P(\text{King} | \text{Ace}) = \frac{4}{52} \cdot \frac{4}{51} = \frac{4}{663}$

   c. Find $P(\text{King on 1st card } \textit{and} \text{ Ace on 2nd})$.

      $P(\text{King}) \cdot P(\text{Ace} | \text{King}) = \frac{4}{663}$

   d. Find the probability of drawing an Ace and a King in either order.

      $\frac{4}{663} + \frac{4}{663} = \frac{8}{663}$

9. You draw two cards from a standard deck of 52 cards, but before you draw the second card, you pu the first one back and reshuffle the deck.

   a. Are the outcomes on the two cards independent? Why?

      Yes, the 1st draw is replaced so the sample space and probabilities do not change.

   b. Find $P(\text{Ace on 1st card } \textit{and} \text{ King on 2nd})$.

      $P(1^{st}\text{ Ace}) \cdot P(2^{nd}\text{ King}) = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$

   c. Find $P(\text{King on 1st card } \textit{and} \text{ Ace on 2nd})$.

      $P(1^{st}\text{ King}) \cdot P(2^{nd}\text{ Ace}) = \frac{1}{169}$

   d. Find the probability of drawing an Ace and a King in either order.

      $P(\text{Ace}, \text{King}) + P(\text{King}, \text{Ace})$

      $= \frac{1}{169} + \frac{1}{169} = \frac{2}{169}$