

In the following problems, (a) find ALL the zeros of the function, and (b) write the polynomial as a product of linear factors (EX: $x^2 - 5x - 6 = (x - 3)(x - 2)$). You might have to use the Quadratic Formula if you can't factor.

1. $h(x) = x^2 + 10x + 23$

a) $x^2 + 10x + 23 = 0$

$$x = \frac{-10 \pm \sqrt{100 - 92}}{2}$$

$$= \frac{-10 \pm 2\sqrt{2}}{2}$$

$$\boxed{= -5 \pm \sqrt{2}}$$

b) $h(x) = [x - (-5 + \sqrt{2})][x - (-5 - \sqrt{2})]$

2. $g(x) = 16x^4 - 81$

a) $(4x^2 - 9)(4x^2 + 9) = 0$

$$\boxed{x = \pm \frac{3}{2}, \pm \frac{3}{2}i}$$

b) $g(x) = (x - \frac{3}{2})(x + \frac{3}{2})(x - \frac{3}{2}i)(x + \frac{3}{2}i)$
OR

$$g(x) = (2x - 3)(2x + 3)(2x - 3i)(2x + 3i)$$

3. $f(x) = 2x^2 + 288$

a) $x^2 + 144 = 0$

$$\boxed{x = \pm 12i}$$

b) $f(x) = 2(x - 12i)(x + 12i)$

4. $k(x) = 3x^3 - 5x^2 + 48x - 80$

a) $x^2(3x - 5) + 16(3x - 5) = 0$

$$(x + 4i)(x - 4i)(3x - 5) = 0$$

$$x = \pm 4i, \frac{5}{3}$$

b) $k(x) = (x + 4i)(x - 4i)(x - \frac{5}{3})$
OR

$$k(x) = (x + 4i)(x - 4i)(3x - 5)$$

5. Find a fourth degree polynomial that has 2, 3, and $-3i$ as zeros.

$$p(x) = (x - 2)(x - 3)(x - 3i)(x + 3i)$$

$$= (x^2 - 5x + 6)(x^2 + 9)$$

$$= x^4 + 9x^2 - 5x^3 - 45x + 6x^2 + 54$$

$$\boxed{= x^4 - 5x^3 + 15x^2 - 45x + 54}$$

6. Find a third degree polynomial that has 2 and $1 - i\sqrt{2}$ as zeros.

$$p(x) = (x - 2)[x - (1 - i\sqrt{2})][x - (1 + i\sqrt{2})]$$

$$= (x - 2)[x^2 - x - xi\sqrt{2} - x + xi\sqrt{2} + 3]$$

$$= (x - 2)[x^2 - 2x + 3]$$

$$= x^3 - 2x^2 + 3x - 2x^2 + 4x - 6$$

$$\boxed{= x^3 - 4x^2 + 7x - 6}$$

Use the given zero to find all the zeros of the function.

7. $f(x) = 2x^3 + 3x^2 + 50x + 75$

Zero: $5i$

$$\begin{array}{r|rrrr} 5i & 2 & 3 & 50 & 75 \\ & & 10i & -50+15i & -75 \\ \hline -5i & 2 & 3+10i & 15i & 0 \\ & & -10i & -15i & \\ \hline & 2 & 3 & 0 & \end{array}$$

$2x + 3 = 0$

$x = -\frac{3}{2}$

$x = \pm 5i, -\frac{3}{2}$

Sketch the following functions by (a) applying the Leading Coefficient Test, (b) finding all real zeros (x-intercepts), (c) plotting a few extra points, and (d) drawing the curve.

8. $f(x) = x^3 + 4x^2 + 5x + 20$

a) L.C. $\rightarrow 1$
exp \rightarrow odd \Rightarrow \uparrow
 \downarrow

b) $x^2(x+4) + 5(x+4) = 0$

$(x+4)(x^2+5) = 0$

$x = -4$

c)

| x | f(x) |
|----|------|
| -1 | 18 |
| 0 | 20 |
| 1 | 30 |

