Name Answer Key

PreCalculus; Sec 2.5a – Fundamental Theorem of Algebra

In the following problems, (a) find ALL the zeros of the function, and (b) write the polynomial as a product of linear factors (EX: $x^2 - 5x - 6 = (x - 3)(x - 2)$). You might have to use the Quadratic Formula if you can't factor.

1. $h(x) = x^2 - 4x + 1$

a) $x^2 - 4x + 1 = 0$

$x = \frac{4 \pm \sqrt{16-4}}{2}$

$= \frac{4 \pm 2\sqrt{3}}{2}$

$\boxed{= 2 \pm \sqrt{3}}$

B) $h(x) = [x - (2+\sqrt{3})][x - (2-\sqrt{3})]$

2. $g(x) = x^2 + 6x - 2$

a) $x^2 + 6x - 2 = 0$

$x^2 + 6x = 2$

$(x+3)^2 = 11$

$\boxed{x = -3 \pm \sqrt{11}}$

B) $g(x) = [x - (-3+\sqrt{11})][x - (-3-\sqrt{11})]$

3. $f(x) = x^2 + 36$

a) $x^2 + 36 = 0$

$x^2 = -36$

$\boxed{x = \pm 6i}$

B) $f(x) = (x - 6i)(x + 6i)$

4. $k(x) = 81x^4 - 625$

a) $81x^4 - 625 = 0$

$(9x^2 - 25)(9x^2 + 25) = 0$

$\boxed{x = \pm \frac{5}{3}, \pm \frac{5}{3}i}$

B) $k(x) = (x + \frac{5}{3})(x - \frac{5}{3})(x + \frac{5}{3}i)(x - \frac{5}{3}i)$

OR

$= (3x + 5)(3x - 5)(3x + 5i)(3x - 5i)$

5. $f(x) = x^4 + 25x^2 + 100$

a) $x^4 + 25x^2 + 100 = 0$

$(x^2 + 5)(x^2 + 20) = 0$

$\boxed{x = \pm i\sqrt{5}, \pm 2i\sqrt{5}}$

B) $f(x) = (x - i\sqrt{5})(x + i\sqrt{5})$

$(x + 2i\sqrt{5})(x - 2i\sqrt{5})$

6. $g(x) = x^4 - x^2 - 56$

a) $x^4 - x^2 - 56 = 0$

$(x^2 - 8)(x^2 + 7) = 0$

$\boxed{x = \pm 2\sqrt{2}, \pm i\sqrt{7}}$

B) $g(x) = (x + 2\sqrt{2})(x - 2\sqrt{2})$

$(x + i\sqrt{7})(x - i\sqrt{7})$

7. Find a fourth degree polynomial that has $-1, 2$, and i as zeros.

$P(x) = (x+1)(x-2)(x+i)(x-i)$

$= (x^2 - x - 2)(x^2 + 1)$

$= x^4 + x^2 - x^3 - x - 2x^2 - 2$

$\boxed{= x^4 - x^3 - x^2 - x - 2}$

8. Find a third degree polynomial that has -1 and $2+\sqrt{5}i$ as zeros.

$P(x) = (x+1)[x-(2+\sqrt{5})][x-(2-\sqrt{5})]$

$= (x+1)[x^2 - 2x + x\sqrt{5} - 2x - x\sqrt{5} + (2+\sqrt{5})(2-\sqrt{5})]$

$= (x+1)[x^2 - 4x + 9]$

$= x^3 - 4x^2 + 9x + x^2 - 4x + 9$

$\boxed{= x^3 - 3x^2 + 5x + 9}$

9. Find a third degree polynomial that has -2 and $2+2\sqrt{2}i$ as zeros.

$P(x) = (x+2)[x-(2+2\sqrt{2}i)][x-(2-2\sqrt{2}i)]$

$= (x+2)[x^2 - 2x - 2x\sqrt{2}i - 2x - 2x\sqrt{2}i + (2+2\sqrt{2}i)(2-2\sqrt{2}i)]$

$= (x+2)[x^2 - 4x + 12]$

$= x^3 - 4x^2 + 12x + 2x^2 - 8x + 24$

$\boxed{= x^3 - 2x^2 + 4x + 24}$

Sketch the following functions by (a) applying the Leading Coefficient Test, (b) finding all real zeros (x-intercepts), (c) plotting a few extra points, and (d) drawing the curve.

10. $f(x) = 3x^3 - 15x^2$

a) L.C. → 3 ⇒ ↗
Exp → ODD ↘

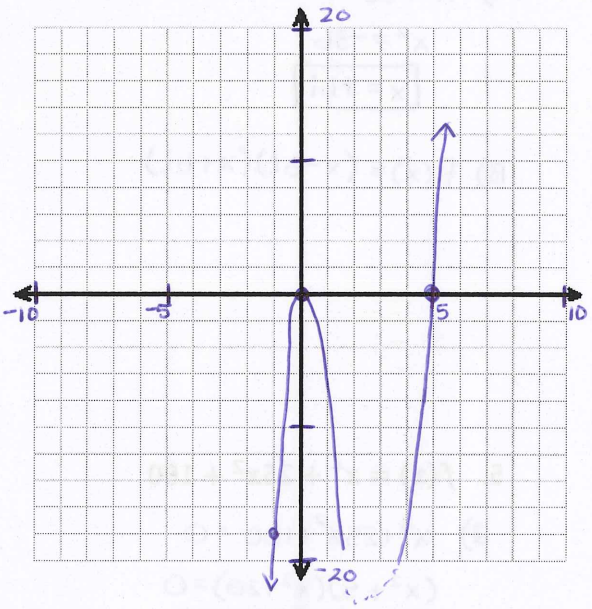
B) x-INTERCEPTS ($y=0$)

$3x^2(x-5) = 0$

$(0,0)$ $(5,0)$

c)

x	y
-1	-18
2	-36



11. $f(x) = -x^4 + 9x^2 - 20$

a) L.C. → -1
Exp → EVEN

B) x-INTERCEPTS ($y=0$)

$x^4 - 9x^2 + 20 = 0$

$(x^2-4)(x^2-5) = 0$

$(\pm 2, 0)$ $(\pm\sqrt{5}, 0)$

c)

x	y
0	-20