

1. Two forms of the Division Algorithm are shown below. Identify and label each part.

$$f(x) = d(x)q(x) + r(x)$$

Labels: $f(x)$ is DIVIDEND, $d(x)$ is DIVISOR, $q(x)$ is QUOTIENT, $r(x)$ is REMAINDER.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

Labels: $f(x)$ is DIVIDEND, $d(x)$ is DIVISOR, $q(x)$ is QUOTIENT, $\frac{r(x)}{d(x)}$ is REMAINDER.

2. The rational expression $\frac{p(x)}{q(x)}$ is called improper when the degree of the numerator is greater than or equal to that of the denominator.

3. Every rational zero of a polynomial function with integer coefficients has the form $\frac{p}{q}$, where p is a factor of the CONSTANT (TERM) and q is a factor of the LEADING COEFFICIENT.

4. You divide the polynomial $f(x)$ by $(x - 4)$ and get an answer of 7. What is $f(4)$?

$$f(4) = 7$$

Use either long division or synthetic division to divide.

5. Divide $x^3 - 4x^2 - 17x + 6$ by $x - 3$

$$\begin{array}{r|rrrr} 3 & 1 & -4 & -17 & 6 \\ & & 3 & -3 & -60 \\ \hline & 1 & -1 & -20 & -54 \end{array}$$

$$\boxed{x^2 - x - 20 - \frac{54}{x-3}}$$

6. $(5x^3 + 6x + 8) \div (x + 2)$

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$

$$\boxed{5x^2 - 10x + 26 - \frac{44}{x+2}}$$

Use the Remainder Theorem and synthetic division to evaluate the function at each given value.

7. $f(x) = 4x^4 - 16x^3 + 7x^2 + 20$

a. $f(1)$

$$\begin{array}{r|rrrrr} 1 & 4 & -16 & 7 & 0 & 20 \\ & & 4 & -12 & -5 & -5 \\ \hline & 4 & -12 & -5 & -5 & 15 \end{array}$$

c. $f(5)$

$$\begin{array}{r|rrrrr} 5 & 4 & -16 & 7 & 0 & 20 \\ & & 20 & 20 & 135 & 675 \\ \hline & 4 & 4 & 27 & 135 & 695 \end{array}$$

b. $f(-2)$

$$\begin{array}{r|rrrrr} -2 & 4 & -16 & 7 & 0 & 20 \\ & & -8 & 48 & -110 & 220 \\ \hline & 4 & -24 & 55 & -110 & 240 \end{array}$$

d. $f(-10)$

$$\begin{array}{r|rrrrr} -10 & 4 & -16 & 7 & 0 & 20 \\ & & -40 & 560 & -5670 & 56700 \\ \hline & 4 & -56 & 567 & -5670 & 56720 \end{array}$$

In Exercise 8, (a) verify the given factor(s) of the function f , (b) find the remaining factors of f , (c) write the complete factorization of f , and (d) list all real zeros of f .

8. $f(x) = 6x^3 + 41x^2 - 9x - 14$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 6 & 41 & -9 & -14 \\ & & -3 & -19 & 14 \\ \hline & 6 & 38 & -28 & 0 \end{array}$$

B) $6x^2 + 38x - 28$
 $2(3x^2 + 19x - 14)$

$2(3x-2)(x+7)$

C) $f(x) = 2(3x-2)(x+7)(x+\frac{1}{2})$

D) $x = \frac{2}{3}, -7, -\frac{1}{2}$

Factors: $(2x + 1)$

$$\begin{array}{r} 3x^2 + 19x - 14 \\ 2x+1 \overline{) 6x^3 + 41x^2 - 9x - 14} \\ \underline{-(6x^3 + 3x^2)} \\ 38x^2 - 9x - 14 \\ \underline{-(38x^2 + 19x)} \\ -28x - 14 \\ \underline{-(-28x - 14)} \\ 0 \end{array}$$

B) $3x^2 + 19x - 14$

$(3x-2)(x+7)$

C) $f(x) = (2x+1)(3x-2)(x+7)$

D) $x = -\frac{1}{2}, \frac{2}{3}, -7$

OR

Find all real zeros of the polynomial function. (Hint: you may need to use the Rational Zero Test.)

9. $h(x) = x^5 - x^4 - 3x^3 + 5x^2 - 2x$

$x(x^4 - x^3 - 3x^2 + 5x - 2) = 0$

$x = 0$

$\frac{p}{q} = \pm \frac{1, 2}{1}$

$$\begin{array}{r|rrrrr} -2 & 1 & -1 & -3 & 5 & -2 \\ & & -2 & 6 & -6 & 2 \\ \hline 1 & 1 & -3 & 3 & -1 & 0 \\ & & 1 & -2 & 1 & \\ \hline & 1 & -2 & 1 & 0 & \end{array}$$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$