

Use either long division or synthetic division to divide.

1. Divide $8x^4 - 5$ by $2x + 1$

$$\begin{array}{r} 4x^3 - 2x^2 + x - 0.5 \\ 2x+1 \overline{) 8x^4 + 0x^3 + 0x^2 + 0x - 5} \\ \underline{-(8x^4 + 4x^3)} \\ -4x^3 + 0x^2 + 0x - 5 \\ \underline{-(-4x^3 - 2x^2)} \\ 2x^2 + 0x - 5 \\ \underline{-(2x^2 + x)} \\ -x - 5 \\ \underline{-(-x - 0.5)} \\ -4.5 \end{array}$$

$4x^3 - 2x^2 + x - 0.5 - \frac{4.5}{2x+1}$

2. Divide $2x^3 - 3x^2 - 50x + 75$ by $2x - 3$

$$\begin{array}{r} x^2 - 25 \\ 2x-3 \overline{) 2x^3 - 3x^2 - 50x + 75} \\ \underline{-(2x^3 - 3x^2)} \\ -50x + 75 \\ \underline{-(-50x + 75)} \\ 0 \end{array}$$

$x^2 - 25$

3. $(5x^2 - 17x - 12) \div (x - 4)$

$$\begin{array}{r} 5 - 17 - 12 \\ 4 \overline{) 5 - 17 - 12} \\ \underline{4 - 20 12} \\ 5 3 0 \end{array}$$

$5x + 3$

4. $(x^3 - 729) \div (x - 9)$

$$\begin{array}{r} 1 0 - 729 \\ 9 \overline{) 1 0 - 729} \\ \underline{9 81 729} \\ 1 9 81 0 \end{array}$$

$x^2 + 9x + 81$

5. $\frac{2x^3 - 4x^2 - 15x + 5}{(x-1)^2}$

$$\begin{array}{r} 2 - 4 - 15 5 \\ 1 \overline{) 2 - 4 - 15 5} \\ \underline{2 - 2 - 17 - 12} \\ 2 0 \\ \underline{2 0 - 17} \end{array}$$

$$\begin{array}{r} 2x \\ x^2 - 2x + 1 \overline{) 2x^3 - 4x^2 - 15x + 5} \\ \underline{-(2x^3 - 4x^2 + 2x)} \\ -17x + 5 \end{array}$$

$2x + \frac{-17x + 5}{(x-1)^2}$

6. $\frac{4x^3 + 16x^2 - 23x - 15}{x + \frac{1}{2}}$

$$\begin{array}{r} 4 16 - 23 - 15 \\ -\frac{1}{2} \overline{) 4 16 - 23 - 15} \\ \underline{-2 - 7 15} \\ 4 14 - 30 0 \end{array}$$

$4x^2 + 14x - 30 = 2x^2 + 7x - 15$

$$2x + \frac{-17(x-1) - 12}{(x-1)^2} = 2x + \frac{-17x + 5}{(x-1)^2}$$

Use the Remainder Theorem and synthetic division to evaluate the function at each given value.

7. $f(x) = 2x^3 - 7x + 3$

- a. $f(1)$

$$\begin{array}{r} 2 0 - 7 3 \\ 1 \overline{) 2 0 - 7 3} \\ \underline{2 2 - 5} \\ 2 2 - 5 - 2 \end{array}$$

- b. $f(-2)$

$$\begin{array}{r} 2 0 - 7 3 \\ -2 \overline{) 2 0 - 7 3} \\ \underline{-4 8 - 2} \\ 2 -4 1 1 \end{array}$$

- c. $f(\frac{1}{2})$

$$\begin{array}{r} 2 0 - 7 3 \\ \frac{1}{2} \overline{) 2 0 - 7 3} \\ \underline{1 \frac{1}{2} - 3.25} \\ 2 1 - 6.5 - 0.25 \end{array}$$

- d. $f(2)$

$$\begin{array}{r} 2 0 - 7 3 \\ 2 \overline{) 2 0 - 7 3} \\ \underline{4 8 2} \\ 2 4 1 5 \end{array}$$

In Exercise 8, (a) verify the given factor(s) of the function f , (b) find the remaining factors of f , (c) write the complete factorization of f , and (d) list all real zeros of f .

8. $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$

Factors: $(x - 5), (x + 4)$

$$\begin{array}{r|rrrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline -4 & 1 & 1 & -10 & 8 & 0 \\ & & -4 & 12 & -8 & \\ \hline & 1 & -3 & 2 & 0 & \end{array}$$

REMAINDER OF 0;
 $x-5$ IS A FACTOR!

B) $x^2 - 3x + 2$
 $(x-1)(x-2)$

C) $f(x) = (x-1)(x-2)(x-5)(x+4)$

D) $x = 1, 2, 5, -4$

Find all real zeros of the polynomial function. (Hint: you may need to use the Rational Zero Test.)

9. $f(x) = 3x^4 - 14x^2 - 4x$

$f(x) = x(3x^3 - 14x - 4)$

$x = 0$

$\frac{P}{Q} = \pm \frac{1, 2, 4}{1, 3}$

$$\begin{array}{r|rrrr} -2 & 3 & 0 & -14 & -4 \\ & & -6 & 12 & 4 \\ \hline & 3 & -6 & -2 & 0 \end{array}$$

$3x^2 - 6x - 2 = 0$

$x = \frac{6 \pm \sqrt{36 + 24}}{6}$

$= \frac{6 \pm 2\sqrt{15}}{6}$

$= \frac{3 \pm \sqrt{15}}{3}$

10. $g(x) = 4x^4 - 11x^3 - 22x^2 + 8x$

$g(x) = x(4x^3 - 11x^2 - 22x + 8)$

$x = 0$

$\frac{P}{Q} = \pm \frac{1, 2, 4, 8}{1, 2, 4}$

$$\begin{array}{r|rrrr} 4 & 4 & -11 & -22 & 8 \\ & & 16 & 20 & -8 \\ \hline & 4 & 5 & -2 & 0 \end{array}$$

$4x^2 + 5x - 2 = 0$

$x = \frac{-5 \pm \sqrt{25 + 32}}{8}$

$= \frac{-5 \pm \sqrt{57}}{8}$