Name Answer Key

PreCalculus; Sec 2.2b – Polynomial Functions of Higher Degree

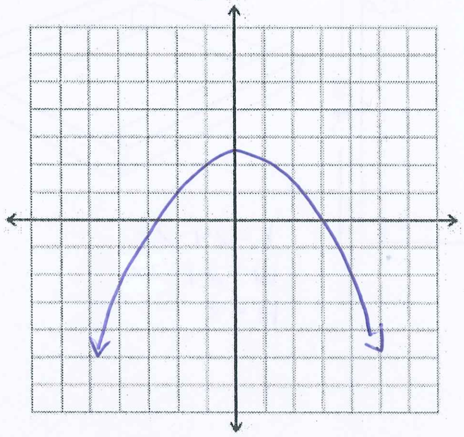
1. The graph of a polynomial function is Continuous, so it has no breaks, holes, or gaps.

2. A polynomial function of degree $n$ has at most N real zeros and at most $n-1$ relative extrema.

3. If a zero of a polynomial function $f$ is of even multiplicity, then the graph of $f$ TOUCHES the x-axis, and if the zero is of odd multiplicity, then the graph of $f$ CROSSES the x-axis.

**Use the Leading Coefficient Test to determine the right and left hand behavior of the polynomial function, then sketch what the graph might look like.**

4. $f(x) = 5 - \frac{7}{2}x - 3x^2$

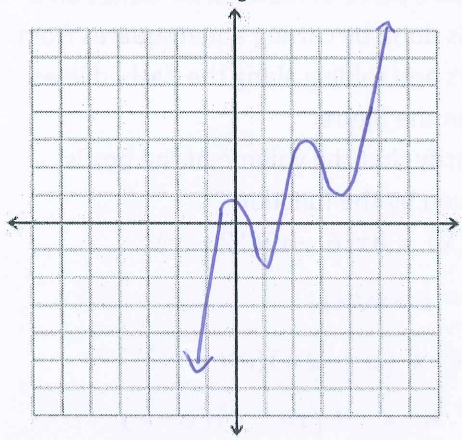


L.C. → $-3$

exp. → even ↙ ↘

Possible extrema 1

5. $g(x) = \frac{6x^5 - 2x^4 + 4x^2 - 5x}{3}$



L.C. → 2 ↗

exp → ODD ↙

Possible extrema 4

**Find all the real zeros of the polynomial function.**

6. $f(t) = \frac{1}{2}t^4 - \frac{1}{2}$

$\frac{1}{2}t^4 - \frac{1}{2} = 0$

$t^4 - 1 = 0$

$t^4 = 1$

$\boxed{t = \pm 1}$

7. $g(x) = \frac{1}{4}x^3(x^2 - 9)$

$\frac{1}{4}x^3(x^2-9) = 0$

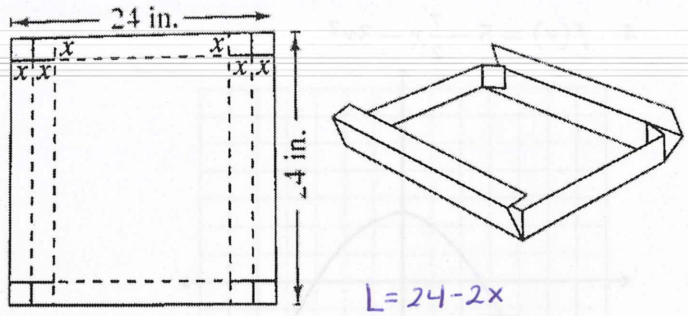
$\boxed{x = 0, \pm 3}$

**Find a polynomial function with the given zeros, multiplicities, and degree.**

8. Zero: $-1$, multiplicity: 2
   Zero: $-2$, multiplicity: 1
   Degree: 3
   Rises to the left, Falls to the right

$$P(x) = -(x+1)^2(x+2)$$
$$= -(x+2)(x^2+2x+1)$$
$$= -x^3-2x^2-x-2x^2-4x-1$$
$$\boxed{= -x^3-4x^2-5x-1}$$

9. An open box with locking tabs is to be made from a square piece of material 24 inches on a side. This is done by cutting equal squares from the corners and folding along the dashed lines, as shown in the figure.




$L = 24-2x$
$W = 24-4x$
$H = x$

   a. Verify that the volume of the box is given by the function $V(x) = 8x(6-x)(12-x)$.

$$V = L \times W \times H$$
$$= x\,(24-2x)(24-4x)$$
$$= x \cdot 2 \cdot 4\,(12-x)(6-x)$$
$$= 8x(12-x)(6-x)\ \checkmark$$

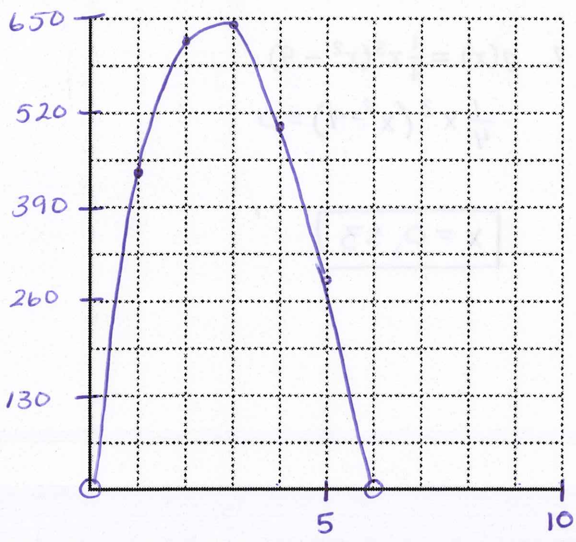
   b. Determine the domain of the function $V$.

$D: 0 < x < 6$

$(0, 6)$

$\{x \mid 0 < x < 6\}$

   c. Sketch the graph of the function and estimate the value of $x$ for which $V(x)$ is maximum.




| x | V(x) |
|---|---|
| 0 | 0 |
| 1 | 440 |
| 2 | 640 |
| 3 | 648 |
| 4 | 512 |
| 5 | 280 |
| 6 | 0 |

$\boxed{\text{MAXIMUM VOLUME AT } x = 3 \text{ in.}}$