Name ANSWER KEY

PreCalculus; Sec 1.6b – Inverse Functions

1. If f and g are functions such that $f(g(x)) = x$ and $g(f(x)) = x$, then the function g is the INVERSE function of f, and is denoted by $f^{-1}(x)$.

2. The domain of f is the RANGE of f^{-1}, and the DOMAIN of f^{-1} is the range of f.

3. The graphs of f and f^{-1} are reflections of each other over the line $y = x$.

4. To have an inverse function, a function f must be ONE-to-ONE; that is $f(a) = f(b)$ implies $a = b$.

Show that f and g are inverse functions algebraically.

5. $f(x) = -\frac{7}{2}x - 3,\ g(x) = -\frac{2x+6}{7}$

$$f(g(x)) = -\frac{7}{2}\left[-\frac{2x+6}{7}\right] - 3$$
$$= \frac{1}{2}\left[2x+6\right] - 3$$
$$= x + 3 - 3$$
$$= x \checkmark$$
$$g(f(x)) = \frac{-2\left[-\frac{7}{2}x - 3\right] + 6}{7}$$
$$= \frac{7x - 6 + 6}{7} = \frac{7x}{7} = x \checkmark$$

6. $f(x) = \frac{x^3}{2},\ g(x) = \sqrt[3]{2x}$

$$f(g(x)) = \frac{(\sqrt[3]{2x})^3}{2} = x \checkmark$$
$$g(f(x)) = \sqrt[3]{2\left(\frac{x^3}{2}\right)} = x \checkmark$$

Determine algebraically whether the function is one-to-one. If so, find its inverse.

7. $g(x) = x^2 - x^4$

$$g(a) = g(B)$$
$$a^2 - a^4 = b^2 - b^4$$
$$a^2(1+a)(1-a) = B^2(1-B)(1+B)$$
$$\boxed{\text{NOT ONE-TO-ONE}}$$

8. $f(x) = (x+3)^2,\ x \geq -3$

$$f(a) = f(B)$$
$$(a+3)^2 = (B+3)^2$$
$$a + 3 = B + 3$$
$$a = B \checkmark$$
one-to-one

$$x = (y+3)^2$$
$$\sqrt{x} = y + 3$$
$$y = \sqrt{x} - 3$$
$$\boxed{f^{-1}(x) = \sqrt{x} - 3,\ x \geq 0}$$

9. $f(x) = \sqrt{2x+3}$

$$f(a) = f(B)$$
$$\sqrt{2a+3} = \sqrt{2b+3}$$
$$2a + 3 = 2b + 3$$
$$2a = 2b$$
$$a = b \checkmark$$
one-to-one

$$x = \sqrt{2y+3}$$
$$x^2 = 2y + 3$$
$$2y = x^2 - 3$$
$$y = \frac{x^2-3}{2}$$
$$\boxed{f^{-1}(x) = \frac{x^2-3}{2}}$$

10. $f(x) = \frac{x^2}{x^2+1}$

$$f(a) = f(b)$$
$$\frac{a^2}{a^2+1} = \frac{B^2}{B^2+1}$$
$$a^2B^2 + a^2 = a^2B^2 + B^2$$
$$a^2 = B^2$$
$$a = B \checkmark$$
ONE-TO-ONE

$$x = \frac{y^2}{y^2+1}$$
$$xy^2 + x = y^2$$
$$x = y^2(1-x)$$
$$y^2 = \frac{x}{1-x}$$
$$y = \sqrt{\frac{x}{1-x}}$$
$$\boxed{f^{-1}(x) = \sqrt{\frac{x}{1-x}}}$$