Name Answer Key

PreCalculus; Sec 1.6a – Inverse Functions

Show that f and g are inverse functions algebraically.

1. $f(x) = x^3;\ g(x) = \sqrt[3]{x}$

$f(g(x)) = \left[\sqrt[3]{x}\right]^3 = x$

$g(f(x)) = \sqrt[3]{x^3} = x$

2. $f(x) = \sqrt{x-4};\ g(x) = x^2 + 4,\ x \geq 0$

$f(g(x)) = \sqrt{(x^2+4)-4}$

$= \sqrt{x^2} = x$

$g(f(x)) = (\sqrt{x-4})^2 + 4$

$= (x-4) + 4 = x$

3. $f(x) = 2x;\ g(x) = \frac{x}{2}$

$f(g(x)) = 2\left(\frac{x}{2}\right) = x$

$g(f(x)) = \frac{2x}{2} = x$

4. $f(x) = \frac{x-1}{x+5};\ g(x) = -\frac{5x+1}{x-1}$

$f(g(x)) = \dfrac{\left[-\frac{5x+1}{x-1}\right] - 1}{\left[-\frac{5x+1}{x-1}\right] + 5} = \dfrac{-(5x+1)-(x-1)}{-(5x+1)+5(x-1)}$

$= \dfrac{-5x-1-x+1}{-5x-1+5x-5} = \dfrac{-6x}{-6} = x$

$g(f(x)) = \dfrac{-\left[5\left[\frac{x-1}{x+5}\right]+1\right]}{\left[\frac{x-1}{x+5}\right]-1}$

$= \dfrac{-[5(x-1)+(x+5)]}{(x-1)-(x+5)} = \dfrac{-5x+5-x-5}{x-1-x-5} = \dfrac{-6x}{-6} = x$

Determine algebraically whether the function is one-to-one. If so, find its inverse.

5. $f(x) = x^4$

$f(a) = f(B)$

$a^4 = B^4$

$\sqrt[4]{a^4} = \sqrt[4]{B^4}$

$a = B$ ✓

$x = y^4$

$y = \sqrt[4]{x}$

$\boxed{f^{-1}(x) = \sqrt[4]{x}}$

6. $h(x) = \frac{1}{x^2}$

$h(a) = h(B)$

$\frac{1}{a^2} = \frac{1}{B^2}$

$a^2 = B^2$

$a = B$ ✓

$x = \frac{1}{y^2}$

$y^2 = \frac{1}{x}$

$y = \frac{1}{\sqrt{x}}$

$\boxed{h^{-1}(x) = \frac{1}{\sqrt{x}}}$

7. $g(x) = \sqrt{x-2}$

$g(a) = g(B)$

$\sqrt{a-2} = \sqrt{B-2}$

$a - 2 = B - 2$

$a = B$ ✓

$x = \sqrt{y-2}$

$x^2 = y - 2$

$y = x^2 + 2$

$\boxed{g^{-1}(x) = x^2 + 2}$

8. $p(x) = \frac{x^2}{x^2+1}$

$P(a) = P(B)$

$\frac{a^2}{a^2+1} = \frac{B^2}{B^2+1}$

$a^2(B^2+1) = B^2(a^2+1)$

$a^2B^2 + a^2 = a^2B^2 + B^2$

$a^2 = B^2$

$a = B$ ✓

$x = \frac{y^2}{y^2+1}$

$xy^2 + x = y^2$

$x = y^2(1-x)$

$y^2 = \frac{x}{1-x}$

$y = \frac{\sqrt{x}}{\sqrt{1-x}}$

$\boxed{p^{-1}(x) = \sqrt{\frac{x}{1-x}}}$

9. You can encode and decode messages using functions and their inverses. To code a message, first translate the letters to numbers using 1 for "A", 2 for "B", and so on. Use 0 for a space. So, "A ball" becomes

1 0 2 1 12 12.

Then, use a one-to-one function to convert to coded numbers. Using $f(x) = 2x - 1$, "A ball" becomes

1 -1 3 1 23 23.

(a) Encode "Do your homework" using the function $f(x) = 5x + 4$.

	D	O	_	Y	O	U	R	_	H	O	M	E	W	O	R	K
ORIGINAL:	4	15	0	25	15	21	18	0	8	15	13	5	23	15	18	11

ENCODED: 24 79 4 129 79 109 94 4 44 79 69 29 119 79 94 59

(b) Find the inverse function of $f(x) = 5x + 4$ and use it to decode 119 44 9 104 4 104 49 69 29.

$x = 5y + 4$

$5y = x - 4$

$y = \frac{x-4}{5}$

$f^{-1}(x) = \frac{x-4}{5}$

Decoded:

23	8	1	20	0	20	9	13	5
W	H	A	T	_	T	I	M	E

What time