Name ANSWER KEY

PreCalculus; Sec 1.5b – Combinations of Functions

For each function, find (a) $(f+g)(x)$, (b) $(f-g)(x)$, (c) $(fg)(x)$, and (d) $(f/g)(x)$. Write the domain of f/g in <u>interval notation</u>.

1. $f(x) = \sqrt{x^2-4}$, $g(x) = \frac{x^2}{x^2+1}$

a) $(f+g)(x) = \sqrt{x^2-4} + \frac{x^2}{x^2+1}$

B) $(f-g)(x) = \sqrt{x^2-4} - \frac{x^2}{x^2+1}$

c) $(fg)(x) = \left[\sqrt{x^2-4}\right]\left[\frac{x^2}{x^2+1}\right]$

D) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x^2-4}}{\frac{x^2}{x^2+1}} = \frac{(x^2+1)\sqrt{x^2-4}}{x^2}$

D: $(-\infty, -2] \cup [2, +\infty)$

2. $f(x) = \frac{x}{x+1}$, $g(x) = x^3$

a) $(f+g)(x) = \frac{x}{x+1} + x^3$

B) $(f-g)(x) = \frac{x}{x+1} - x^3$

c) $(fg)(x) = \frac{x^4}{x+1}$

D) $\left(\frac{f}{g}\right)(x) = \frac{\frac{x}{x+1}}{x^3} = \frac{1}{x^2(x+1)}$

D: $(-\infty, -1) \cup (-1, 0) \cup (0, +\infty)$

Evaluate the indicated function for $f(x) = x - 1$ and $g(x) = x^2$.

3. $(f+g)(2)$

$= (2) - 1 + (2)^2$

$= 2 - 1 + 4$

$\boxed{= 5}$

4. $(fg)(4)$

$= (4-1)(4)^2$

$= 3(16)$

$\boxed{= 48}$

Find (a) $f \circ g$, (b) $g \circ f$, and, if possible, (c) $(f \circ g)(0)$.

5. $f(x) = \sqrt[3]{x-1}$, $g(x) = x^3 + 1$

a) $f(g(x)) = \sqrt[3]{x^3+1-1} = x$

B) $g(f(x)) = (\sqrt[3]{x-1})^3 + 1 = x$

c) $f(g(0)) = 0$

6. $f(x) = x^3$, $g(x) = \frac{1}{x}$

a) $f(g(x)) = \frac{1}{x^3}$

B) $g(f(x)) = \frac{1}{x^3}$

c) NO SOLUTION

Determine the domains of (a) f, (b) g, and (c) $f \circ g$.

7. $f(x) = x^2 + 1$, $g(x) = \sqrt{x}$

a) D: $(-\infty, +\infty)$

B) D: $[0, +\infty)$

c) $f(g(x)) = (\sqrt{x})^2 + 1$

D: $[0, +\infty)$

8. $f(x) = x + 2$, $g(x) = \frac{1}{x^2-4}$

a) D: $(-\infty, +\infty)$

B) D: $(-\infty, -2) \cup (-2, 2) \cup (2, +\infty)$

c) $f(g(x)) = \frac{1}{x^2-4} + 2$

D: $(-\infty, -2) \cup (-2, 2) \cup (2, +\infty)$

9. The annual cost C (in thousands of dollars) and revenue R (in thousands of dollars) for a company each year from 2004 through 2010 can be approximated by the models

$$C = 260 - 8t + 1.6t^2 \text{ and } R = 320 + 2.8t$$

where t is the year, with $t = 4$ corresponding to 2004.

(a) Write a function P that represents the annual profit of the company.

$P = R - C$

$P(t) = 60 + 10.8t - 1.6t^2$

(b) How much profit did the company make in 2009?

$t = 9$

$P(9) = 60 + 10.8(9) - 1.6(9)^2$

$\boxed{= \$27.6 \text{ THOUSAND}}$

10. The suggested retail price of a new car is p dollars. The dealership advertised a factory rebate of $1200 and an 8% discount.

(a) Write a function R in terms of p giving the cost of the car after receiving the rebate from the factory.

$R = p - 1200$

(b) Write a function S in terms of p giving the cost of the car after receiving the dealership discount.

$S = 0.92p$

(c) Form the composite functions $(R \circ S)(p)$ and $(S \circ R)(p)$. What do each of these functions mean?

$R(S(p)) = 0.92p - 1200$ ← DISCOUNT, THEN REBATE

$S(R(p)) = 0.92(p - 1200)$ ← REBATE, THEN DISCOUNT

(d) Find $(R \circ S)(18{,}400)$ and $(S \circ R)(18{,}400)$. Which yields the lower cost for the car? Explain.

$R(S(18{,}400) = 0.92(18400) - 1200 = \boxed{\$15{,}728}$ ← DISCOUNT, THEN REBATE GIVES THE LOWER COST.

$S(R(18{,}400) = 0.92(18400 - 1200) = \$15{,}824$