Chapter 7 – Linear Systems

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**Vocabulary**

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<th>Systems of equations</th>
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Section 7.1 Solving Systems of Equations

Objective: In this lesson you learned how to solve systems of equations by substitution and by graphing and how to use systems of equations to model and solve real-life problems.

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<th>Important Vocabulary</th>
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I. The Methods of Substitution and Graphing
To check that the ordered pair \((-3, 4)\) is the solution of a system of two equations, you:

List the steps necessary for solving a system of two equations in \(x\) and \(y\) using the method of substitution:

1. 
2. 
3. 
4. 
5. 

The solution of a system of equations corresponds to the ________________________________ of the graphs of the equations in the system.
To use the method of graphing to solve a system of two equations in $x$ and $y$, perform the following steps:

1. 

2. 

3. 

4. 

Explain what is meant by back-substitution.

II. Points of Intersection and Applications

The total cost $C$ of producing $x$ units of a product typically has two components:

______________________________

In break-even analysis, the break-even point corresponds to the __________________ of the cost and revenue curves.

Break-even analysis can also be approached from the point of view of profit. In this case, consider the profit function, which is ______________. The break-even point occurs when profit equals ______________.
Section 7.1 Examples – Solving Systems of Equations

(1) Solve the system of equations using the method of substitution.

\[
\begin{align*}
2x + y &= 2 \\
(x - 2y) &= -9
\end{align*}
\]

(2) The cost of producing \( x \) units is \( C = 1.5x + 15000 \) and the revenue obtained by selling \( x \) units is \( R = 5x \). How many items should be sold to break even?

(3) Solve the system of equations using the method of graphing.

\[
\begin{align*}
x^2 - y &= 5 \\
-x + y &= -3
\end{align*}
\]

(4) Solve the system of equations graphically.

\[
\begin{align*}
-x + y &= 3 \\
x^2 + y^2 - 6x - 27 &= 0
\end{align*}
\]
Section 7.2 Systems of Linear Equations in Two Variables

Objective: In this lesson you learned how to solve a system of equations by elimination and how to use systems of equations to model and solve real-life problems.

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<td>Method of Elimination</td>
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<td>Inconsistent System</td>
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I. **The Method of Elimination**
   List the steps necessary for solving a system of two linear equations in \( x \) and \( y \) using the method of elimination.
   1. 

   2. 

   3. 

   4. 

   The operations that can be performed on a system of linear equations to produce an equivalent system are:
   1. 

   2. 

   3. 

II. **Graphical Interpretation of Two-Variable Systems**
   If a system of linear equations has two different solutions, it must have __________________________ solutions.

   For a **system of two linear equations in two variables**, list the possible number of solutions the system can have and give a graphical interpretation of the solutions.

What you should learn:

- How to use the method of elimination to solve systems of linear equations in two variables
- How to graphically interpret the number of solutions of a system of linear equations in two variables

Important Vocabulary

- Method of Elimination
- Equivalent Systems
- Consistent System
- Inconsistent System
If a false statement such as $9 = 0$ is obtained while solving a system of linear equations using the method of elimination, then the system has ____________________________.

If a statement that is true for all values of the variables, such as $0 = 0$, is obtained while solving a system of linear equations using the method of elimination, then the system has ____________________________.

III. **Applications of Two-Variable Linear Systems**

When may a system of linear equations be an appropriate mathematical model for solving a real-life application?

What you should learn:

| How to use systems of linear equations in two variables to model and solve real-life problems |
Section 7.2 Examples – Systems of Linear Equations in Two Variables

(1) Describe a strategy for solving the system of linear equations using the method of elimination.
\[
\begin{align*}
3x + y &= 9 \\
4x - 2y &= -1
\end{align*}
\]

(2) Solve the system of linear equations using the method of elimination.
\[
\begin{align*}
4x + y &= -3 \\
x - 3y &= 9
\end{align*}
\]

(3) Solve the system of linear equations using the method of elimination.
\[
\begin{align*}
x - 2y &= 3 \\
-2x + 4y &= 1
\end{align*}
\]

(4) Solve the system of linear equations using the method of elimination.
\[
\begin{align*}
2x - y &= 1 \\
4x - 2y &= 2
\end{align*}
\]
Section 7.3 Multivariable Linear Systems

Objective: In this lesson you learned how to solve a system of equations by Gaussian elimination, how to recognize linear systems in row-echelon form and to use back substitution to solve the system, how to solve nonsquare systems of equations, and how to use a system of equations to model and solve real-life problems.

### Important Vocabulary

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<tr>
<th>Row-Echelon Form</th>
<th>Gaussian Elimination</th>
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<td>Nonsquare System of Equations</td>
<td>Graph of an Equation in Three Variables</td>
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<td>Partial Fraction</td>
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I. **Row-Echelon Form and Back-Substitution**

   When elimination is used to solve a system of linear equations, the goal is:

II. **Gaussian Elimination**

   To solve a system that is not in row-echelon form, you:

   - What you should learn:
     - How to use back-substitution to solve linear systems in row-echelon form
     - How to use Gaussian elimination to solve systems of linear equations

List the three elementary row operations that can be used on a system of linear equations to produce an equivalent system of linear equations.

1.

2.

3.
The number of solution(s) of a system of linear equations in more than two variables must fall into one of the following three categories:

1. 

2. 

3. 

A consistent system having exactly one solution is _______________________________.
A consistent system with infinitely many solutions is _____________________________.

III. **Nonsquare Systems**
In a square system of linear equations, the number of equations in the system is ________________________________ the number of variables.

A system of linear equations cannot have a unique solution unless there are:

IV. **Graphical Interpretation of Three-Variable Systems**
To construct a three-dimensional coordinate system:

To sketch the graph of a plane, you:

The graph of a system of three linear equations in three variables consists of _______________ planes. When these planes intersect in a single point, the system has _______________ solution(s). When the three planes have no point in common, the system has _________ solution(s).
When the planes intersect in a line or a plane, the system has _____________________________ solution(s).
V. Partial Fraction Decomposition

A rational expression can often be written as the sum of two or more simpler rational expressions. For example, the rational expression

\[
\frac{x + 7}{x^2 - x - 6}
\]

can be written as:

Each fraction on the right side of the equation is a _______________________, and together they make up the ______________________ of the left side.

Decomposition of \( \frac{N(x)}{D(x)} \) into Partial Fractions:

1.

2.

3.

4.
Section 7.3 Examples – Multivariable Linear Systems

(1) Solve the system of linear equations.

\[
\begin{align*}
    x + y - z &= 9 \\
    y - 2z &= 4 \\
    z &= 1
\end{align*}
\]

(2) Solve the system of linear equations.

\[
\begin{align*}
    x + y + z &= 3 \\
    2x - y + 3z &= 16 \\
    x - 2y - z &= 1
\end{align*}
\]

(3) The following equivalent system is obtained during the course of Gaussian elimination. Write the solution of the system.

\[
\begin{align*}
    x + 2y - z &= 4 \\
    y + 2z &= 8 \\
    0 &= 0
\end{align*}
\]
(4) Solve the system of linear equations.
\[
\begin{align*}
    x + y + z &= 1 \\
    x - 2y - 2z &= 4
\end{align*}
\]

(5) Solve the basic equation \(5x + 3 = A(x - 1) + B(x + 3)\) for \(A\) and \(B\).

(6) Write the form of the partial fraction decomposition for \(\frac{x-4}{x^2-8x+12}\).