Name ______________________

Chapter 5 Elementary Probability Theory

Section 5.1 What is Probability?

Objective: In this lesson you learned how to work with probabilities and the law of large numbers and apply them to real-life situations.

Important Vocabulary

Probability	Relative Frequency	Equally Likely Outcomes	Law of Large Numbers
Event	Simple Event	Sample Space	Complement of an Event

Focus Points:
- Assign probabilities to events
- Explain how the law of large numbers relates to relative frequencies
- Apply basic rules of probability in everyday life

I. **Probability**

Probability is:

$P(A)$, read "P of A," denotes the **probability of event A**.

1.

2.

Probability Assignments

1.

2.

3.

A **statistical experiment** or:

An **event** is a:

A **simple event** is:

Sample space –

The **sum** of all probabilities = __________

The **complement of event A**:

Some Important Facts About Probability

1.

2.

3.

4.

5.

6.

7.

II. **Interpreting Probabilities**

What does the probability of an event tell us?

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Focus Points:

- Explain the relationship between statistics and probability

Section 5.1 Examples – What is Probability?

(1) Assign a probability to the indicated event on the basis of the information provided. Indicate the technique you used: intuition, relative frequency, or the formula for equally likely outcomes.

a. A random sample of 500 students at Hudson College were surveyed and it was determined that 375 wear glasses or contact lenses. Estimate the probability that a Hudson College student selected at random wears corrective lenses.

b. The Friends of the Library hosts a fundraising barbecue. George is on the cleanup committee. There are four members on this committee, and they draw lots to see who will clean the grills. Assuming that each member is equally likely to be drawn, what is the probability that George will be assigned the grill-cleaning job?

c. Joanna photographs whales for Sea Life Adventure Films. On her next expedition, she is to film blue whales feeding. Based on her knowledge of the habit of blue whales, she is almost certain she will be successful. What specific number do you suppose she estimates for the probability of success?

(2) Professor Gutierrez is making up a final exam for a course in literature of the southwest. He wants the last three questions to be of the true-false type. To guarantee that the answers do not follow his favorite pattern, he lists all possible true-false combinations for three questions on slips of paper and then picks one at random from a hat.

a. Finish listing the outcomes in the given sample space.

T T T	F T T	T F T	________
T T F	F T F	T F F	________

b. What is the probability that all three items will be false? Use the formula

$$P(\text{all F}) = \frac{\text{No. of favorable outcomes}}{\text{Total No. of outcomes}}$$

c. What is the probability that exactly two items will be true?

(3) A veterinarian tells you that if you breed two cream-colored guinea pigs, the probability that an offspring will be pure white is 0.25. What is the probability that an offspring will not be pure white?

a. $P(\text{pure white}) + P(not \text{ pure white}) =$ ___________

b. $P(not \text{ pure white}) =$ ___________

Section 5.2 Some Probability Rules – Compound Events

Objective: In this lesson you will learn to compute probabilities of general compound events, independent events, and compute conditional probabilities.

Important Vocabulary		
Independent Event	Dependent Event	Conditional Probability
Mutually Exclusive	Disjoint	

I. Conditional Probability and Multiplication Rules

Focus Points:
- Compute probabilities of general compound events

Two events are **independent:**

In **dependent events**, the outcome of the first event:

<u>Multiplication Rule for Independent Events</u>

What is **conditional probability**?

<u>General Multiplication Rule for Any Event</u>

<u>Conditional Probability (When $P(B) \neq 0$)</u>

When do the multiplication rules apply?

What word indicates *together*?

How to use the Multiplication Rules

1.

2.

3.

What does conditional probability tell us?

-
-
-
-
-

II. Addition Rules

What is another way to combine events?

Focus Points:

- Compute probabilities involving independent events or mutually exclusive events
- Use survey results to compute conditional probabilities

What are the ways to satisfy the condition $A\ or\ B$?

1.

2.

3.

Two events are **mutually exclusive** or:

<u>Addition Rule for Mutually Exclusive Events $A\ and\ B$</u>

<u>General Addition Rule for any events $A\ and\ B$</u>

How to use the Addition Rules

1.

2.

3.

Summary of Basic Probability Rules

1.

2.

3.

4.

5.

6.

7.

8.

Section 5.2 – Some Probability Rules – Compound Events

(1) Andrew is 55, and the probability that he will be alive in 10 years is 0.72. Ellen is 35, and the probability that she will be alive in 10 years is 0.92. Assuming that the life span of one will have no effect on the life span of the other, what is the probability that they will both be alive in 10 years?

a. Are these events dependent or independent?

b. Use the appropriate multiplication rule to find $P(\text{Andrew alive in 10 years } and \text{ Ellen alive in 10 years})$.

(2) A quality-control procedure for testing Ready-Flash digital cameras consists of drawing two cameras at random from each lot of 100 without replacing the first camera before drawing the second. If both are defective, the entire lot is rejected. Find the probability that both cameras are defective if the lot contains 10 defective cameras. Since we are drawing the cameras at random, assume that each camera in the lot has an equal chance of being drawn.

a. What is the probability of getting a defective camera on the first draw?

b. The first camera is not replaced, so there are only 99 cameras for the second draw. What is the probability of getting a defective camera on the second draw if the first camera was defective?

c. Are the probabilities computed in parts (a) and (b) different? Does drawing a defective camera on the first draw change the probability of getting a defective camera on the second draw? Are the events dependent?

d. Use the formula for dependent events, $P(A\ and\ B) = P(A) \cdot P(B|A)$ to compute $P(\text{1st camera defective } and \text{ 2nd camera defective})$.

(3) Indicate how each of the following pairs of events are combined. Use either the *and* combination or the *or* combination.

a. Satisfying the humanities requirement by taking a course in the history of Japan or by taking a course in classical literature.

b. Buying new tires and aligning the tires.

c. Getting an A not only in psychology but also in biology.

d. Having at least one of the pets: cat, dog, bird, rabbit.

(4) The Cost Less Clothing Store carries remainder pairs of slacks. If you buy a pair of slacks in your regular waist size without trying them on, the probability that the waist will be too tight is 0.30 and the probability that it will be too lose is 0.10.

a. Are the events "too tight" and "too loose" mutually exclusive?

b. If you choose a pair of slacks at random in your regular waist size, what is the probability that the waist will be too tight or too lose?

(5) Professor Jackson is in charge of a program to prepare people for a high school equivalency exam. Records show that 80% of the students need work in math, 70% need work in English, and 55% need work in both areas.

a. Are the events "needs math" and "needs English" mutually exclusive?

b. Use the appropriate formula to compute the probability that a student selected at random needs math *or* needs English.

(6) Using the table to the right, let's consider other probabilities regarding the types of employees at Hopewell and their political affiliations. This time let's consider the production worker and the affiliation of Independent. Suppose an employee is selected at random from the group of 140.

Employee Type and Political Affiliation

	Political Affiliation				
Employee Type	Deomcrat (D)	Republican (R)	Independent (I)	Row Total	
Executive (E)	5	34	9	48	
Production Worker (PW)	63	21	8	92	
Column Total	68	55	17	140	Grand Total

a. Compute $P(I)$ and $P(PW)$.

b. Compute $P(I|PW)$. This is a conditional probability. Be sure to restrict your attention to production workers, since that is the condition given.

c. Compute $P(I\ and\ PW)$. In this case, look at the entire sample space and the number of employees who are both Independent and in production.

d. Use the multiplication rule for dependent events to calculate $P(I\ and\ PW)$. Is the result the same as that of part (c)?

e. Compute $P(I\ or\ PW)$. Are the events mutually exclusive?

Section 5.3 Trees and Counting Techniques

Objective: In this lesson you learned to use tree diagrams, compute number of ordered arrangements, and nonordered groupings.

Important Vocabulary		
Multiplication Rule of Counting	Tree Diagram	Factorial Notation
Permutation	Combination	

Focus Points:

- Organize outcomes in a sample space using tree diagrams
- Compute the number of ordered arrangements of outcomes using permutations
- Compute number of (nonordered) groupings of outcomes using combinations
- Explain how counting techniques relate to probability in everyday life

What formula is used, when outcomes are equally likely, to compute the probability of an event?

Multiplication Rule of Counting

What is a **tree diagram**?

Factorial Notation

Permutations

Combinations

What are the differences between combinations and permutations?

How to determine the number of outcomes of an experiment.

1.

2.

3.

What do counting rules tell us?

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-
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Section 5.3 – Trees and Counting Techniques

(1) Louis played three tennis matches. Use a tree diagram to list the possible win and loss sequences Louis can experience for the set of three matches.

a. On the first match Louis can win or lose. From Start, indicate these two branches.
b. Regardless of whether Louis wins or loses the first match, he plays the second and can again win or lose. Attach branches representing these two outcomes to *each* of the first match results.
c. Louis may win or lose the third match. Attach braches representing these two outcomes to *each* of the second match results.

d. How many possible win-lose sequences are there for the three matches?

e. Complete the list of win-lose sequences.

1st	2nd	3rd
W	W	W
W	W	L
W	L	W
W	L	L
______	______	______
______	______	______
______	______	______
______	______	______

(2) The board of directors at Belford Community Hospital has 12 members.

Three officers – president, vice president, and treasurer – must be elected from among the members. How many different slates of officers are possible? We will view a slate of officers as a list of three people, with the president listed first, the vice president listed second, and the treasurer listed third. Not only are we asking for the number of different groups of three names for a slate, we are also concerned about order.

a. Do we use the permutations rule or the combinations rule? What is the value of n? What is the value of r?

b. Use the permutations rule with $n = 12$ and $r = 3$ to compute ${}_{12}P_3$.

Three members from the group of 12 on the board of directors at Belford Community Hospital will be selected to go to a convention (all expenses paid) in Hawaii. How many different groups of 3 are there?

c. Do we use the permutations rule or the combinations rule? What is the value of n? What is the value of r?

d. Use the combinations rule with $n = 12$ and $r = 3$ to compute ${}_{12}C_3$.