Chapter 4 – Trigonometric Functions

Vocabulary

Angle	Initial Side
Terminal side	Standard Position
Positive angle	Negative angle
Coterminal	Radian
Central angle of a circle	Complementary angle
Supplementary angle	Degree
Unit circle	Sine
Cosine	Tangent
Secant	Cosecant
Cotangent	Period
Hypotenuse	Opposite side
Adjacent side	Angle of elevation
Angle of depression	Reference angle
Amplitude	Inverse sine
Inverse cosine	Inverse tangent
Bearing	

Section 4.1 Radian and Degree Measure

Objective: In this lesson you learned how to describe an angle and to convert between degree and radian measure

Important Vocabulary			
Degree	**Angle**	**Initial Side**	**Terminal Side**
Standard Position	**Positive Angle**	**Negative Angle**	**Coterminal**
Radian	**Central angle of a circle**	**Complementary Angles**	**Supplementary Angles**

I. Angles

An **angle** is determined by:

What you should learn:

How to describe angles

The **initial side** of an angle is:

The **terminal side** of an angle is:

The **vertex** of an angle is:

An angle is in **standard position** when:

A **positive angle** is generated by a(n) ______________________________ rotation; whereas a **negative angle** is generated by a(n) ______________________________ rotation.

If two angles are **coterminal**, then they have:

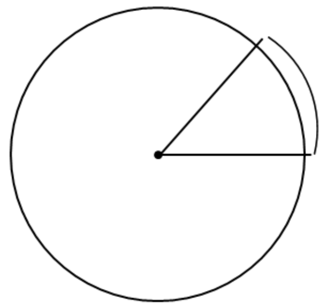
II. Radian Measure

What you should learn:

How to use radian measure

The measure of an angle is determined by:

One **radian** is the measure of a central angle θ that:



Algebraically this means that $\theta =$

A **central angle of one full revolution** (counterclockwise) corresponds to an arc length of

$s =$_________.

The radian measure of an angle one full revolution is _________ radians. A half revolution corresponds to an angle of _________ radians. Similarly $\frac{1}{4}$ revolution corresponds to an angle of _________ radians, and $\frac{1}{6}$ revolution corresponds to an angle of _________ radians.

Angles with measures between 0 and $\frac{\pi}{2}$ radians are ________________ angles. Angles with measures between $\frac{\pi}{2}$ and π radians are ________________ angles.

III. Degree Measure

What you should learn:

How to use degree measure and convert between degrees and radian measure

A full revolution (counterclockwise) around a circle corresponds to _________ degrees. A half revolution around a circle corresponds to _________ degrees.

To convert degrees to radians, you:

To convert radians to degrees, you:

IV. Linear and Angular Speed

What you should learn:

How to use angles to model and solve real-life problems

For a circle of radius r, a central angle θ intercepts an arc f

length s given by ____________ where θ is measured in radians.

Note that if $r = 1$, then $s = \theta$, and the radian measure of θ

equals ______________________________.

Consider a particle moving at a constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed** of the particle is

linear speed $=$______________________________

If θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** of the particle is

angular speed $=$______________________________

Section 4.1 Examples – Radian and Degree Measure

(1) Determine the quadrant in which the angle lies.

a) $55°$ b) $215°$ c) $\frac{\pi}{6}$ d) $\frac{5\pi}{4}$

(2) Sketch the angle in standard position.

a) $45°$ b) $405°$ c) $\frac{3\pi}{4}$ d) $\frac{4\pi}{3}$

(3) Determine two coterminal angles (one positive and one negative) for the given angle.

$$\theta = 35°$$

(4) Convert the angle from degrees to radians.

a) $75°$ b) $-45°$

(5) Convert the angle from radians to degrees.

a) $\frac{2\pi}{3}$ b) $\frac{3\pi}{2}$

(6) Find the length of the arc on a circle of radius r intercepted by a central angle θ.

$$r = 14 \text{ inches}, \theta = 180°$$

Section 4.2 Trigonometric Functions: The Unit Circle

Objective: In this lesson you learned how to identify a unit circle and describe its relationship to real numbers.

Important Vocabulary				
Unit Circle	**Periodic**	**Period**	**Sine**	**Cosine**
Tangent	**Cosecant**	**Secant**	**Cotangent**	

I. The Unit Circle

What you should learn:

How to identify a unit circle and describe its relationship to real numbers

As the real number line is wrapped around the **unit circle**, each real number t corresponds to:

The real number 2π corresponds to the point (_____, _____) on the unit circle.

Each real number t also corresponds to a ______________________________ (in standard position) whose radian measure is t. With this interpretation of t, the arc length formula $s = r\theta$ (with $r = 1$) indicates that:

II. The Trigonometric Functions

What you should learn:

How to evaluate trigonometric functions using the unit circle

The coordinates x and y are two functions of the real variable t. These coordinates can be used to define six trigonometric functions of t. List the abbreviation for each trigonometric function.

Sine	_________	Cosecant	_________
Cosine	_________	Secant	_________
Tangent	_________	Cotangent	_________

Let t be a real number and let (x, y) be the point on the unit circle corresponding to r. Complete the following definitions of the trigonometric functions:

$\sin t =$ ____________ $\cos t =$ ____________

$\tan t =$ ____________ $\cot t =$ ____________

$\sec t =$ ____________ $\csc t =$ ____________

The cosecant function is the reciprocal of the ______________ function. The cotangent function is the reciprocal of the ______________ function. The secant function is the reciprocal of the ______________ function.

Complete the following table showing the correspondence between the real number t and the point (x, y) on the unit circle when the unit circle is divided into eight equal arcs.

t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
x								
y								

Complete the following table showing the correspondence between the real number t and the point (x, y) on the unit circle when the unit circle is divided into 12 equal arcs.

t	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π	$7\pi/6$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$11\pi/6$
x												
y												

III. Domain and Period of Sine and Cosine

What you should learn:

How to use domain and period to evaluate sine and cosine functions

The sine function's domain is ____________________________

and its range is [_____, _____].

The cosine function's domain is

____________________________ and its range is

[_____, _____].

The **period** of the sine function is _______. The period of the cosine function is _______.

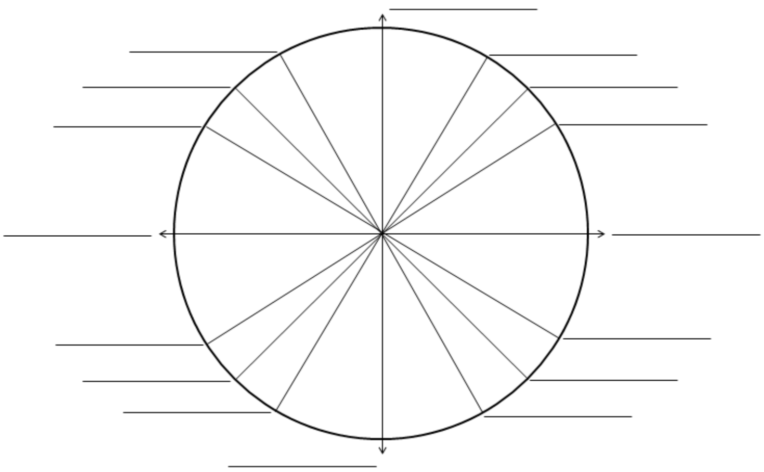
Which trigonometric functions are even functions? ____________________________

Which trigonometric functions are odd functions? ____________________________________

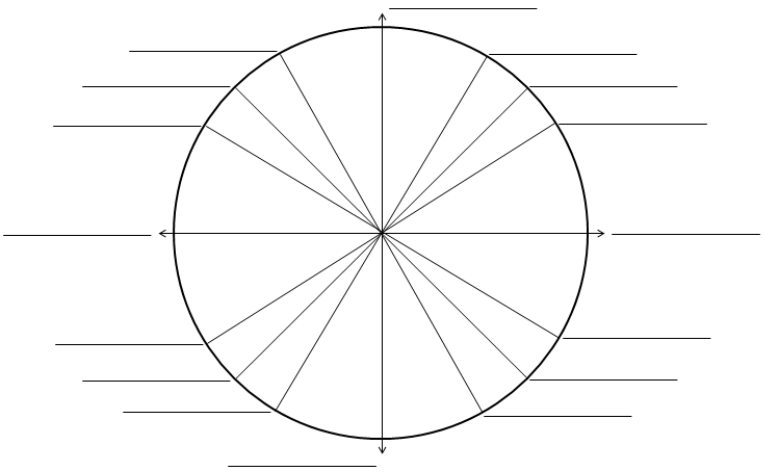
Section 4.2 Examples – Trigonometric Functions: The Unit Circle

(1) Complete the Unit Circles below.

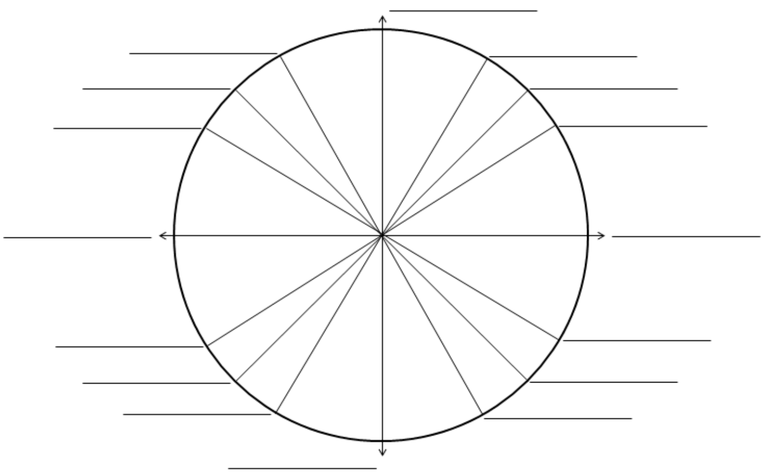
a) Degrees



b) Radians



c) (x, y) values



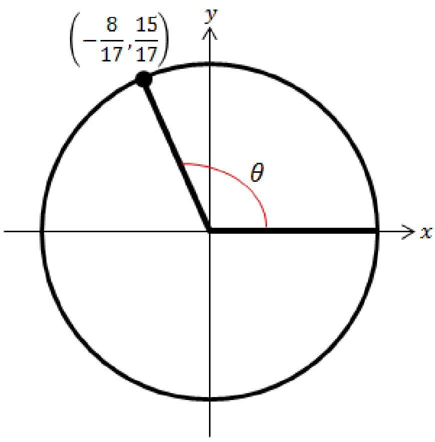
(2) Find the point (x, y) on the unit circle that corresponds to the real number t.

$$t = \frac{5\pi}{4}$$

(3) Evaluate (if possible) the six trigonometric functions of the real number.

$$t = \frac{3\pi}{4}$$

(4) Determine the exact values of the six trigonometric functions of the angle θ.



Section 4.3 Right Triangle Trigonometry

Objective: In this lesson you learned how to evaluate trigonometric functions of acute angles and how to use the fundamental trigonometric identities.

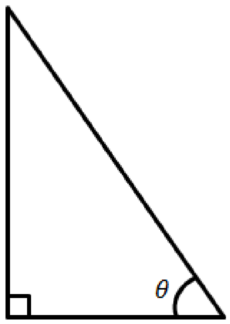
Important Vocabulary		
Hypotenuse	**Opposite Side**	**Adjacent Side**
Angle of Elevation	**Angle of Depression**	

I. The Six Trigonometric Functions

What you should learn:

How to evaluate trigonometric functions of acute angles

In the right triangle below, label the three sides of the triangle relative to the angle labeled θ as (a) the **hypotenuse**, (b) the **opposite side**, and (c) the **adjacent side**.



Let θ be an acute angle of a right triangle. Define the six trigonometric functions of the angle θ using $opp =$ the length of the side opposite θ, $adj =$ the length of the side adjacent to θ, and $hyp =$ the length of the hypotenuse.

$\sin\theta =$ ________________ $\cos\theta =$ ________________

$\tan\theta =$ ________________ $\csc\theta =$ ________________

$\sec\theta =$ ________________ $\cot\theta =$ ________________

The cosecant function is the reciprocal of the ______________ function. The cotangent function is the reciprocal of the ______________ function. The secant function is the reciprocal of the ______________ function.

Give the sines, cosines, and tangents of the following special angles:

$\sin 30° = \sin\frac{\pi}{6} =$ ________ $\cos 30° = \cos\frac{\pi}{6} =$ ________

$\tan 30° = \tan\frac{\pi}{6} =$ ________ $\sin 45° = \sin\frac{\pi}{4} =$ ________

$\cos 45° = \cos\frac{\pi}{4} =$ ________ $\tan 45° = \tan\frac{\pi}{4} =$ ________

$\sin 60° = \sin\frac{\pi}{3} =$ ________ $\cos 60° = \cos\frac{\pi}{3} =$ ________

$\tan 60° = \tan\frac{\pi}{3} =$ ________

Cofunctions of complementary angles are ______________. If θ is an acute angle, then:

$\sin(90° - \theta) =$ ____________ $\cos(90° - \theta) =$ ____________

$\tan(90° - \theta) =$ ____________ $\cot(90° - \theta) =$ ____________

$\sec(90° - \theta) =$ ____________ $\csc(90° - \theta) =$ ____________

II. Trigonometric Identities

What you should learn:

How to use the fundamental trigonometric identities

List six reciprocal identities:

1)

2)

3)

4)

5)

6)

List two quotient identities:

1)

2)

List three Pythagorean identities:

1)

2)

3)

III. Applications Involving Right Triangles

What does it mean to "solve a right triangle?"

What you should learn:

How to use trigonometric functions to model and solve real-life problems

An **angle of elevation** is:

An **angle of depression** is:

Section 4.3 Examples – Right Triangle Trigonometry

(1) Sketch a right triangle corresponding to the trigonometric function of the acute angle θ.

$$\sin\theta = \frac{5}{6}$$

(2) Use the given function value(s) and the trigonometric identities to find the indicated trigonometric functions.

$$\sin 60° = \frac{\sqrt{3}}{2}, \qquad \cos 60° = \frac{1}{2}$$

a. $\tan 60°$

b. $\sin 30°$

c. $\cos 30°$

d. $\cot 60°$

(3) Use identities to transform one side of the equation into the other $\left(0 < \theta < \frac{\pi}{2}\right)$.

$\tan\theta\cot\theta = 1$

Section 4.4 Trigonometric Functions of Any Angle

Objective: In this lesson you learned how to evaluate trigonometric functions of any angle.

Important Vocabulary
Reference Angle

I. Introduction

What you should learn:

How to evaluate trigonometric functions of any angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$. Complete the following definitions of the trigonometric functions of any angle.

$\sin\theta =$ ________________ $\cos\theta =$ ________________

$\tan\theta =$ ________________ $\csc\theta =$ ________________

$\sec\theta =$ ________________ $\cot\theta =$ ________________

Name the quadrant(s) in which the sine function is positive: ______________

Name the quadrant(s)in which the sine function is negative: ______________

Name the quadrant(s)in which the cosine function is positive: ______________

Name the quadrant(s)in which the cosine function is negative: ______________

Name the quadrant(s)in which the tangent function is positive: ______________

Name the quadrant(s)in which the tangent function is negative: ______________

II. Reference Angles

What you should learn:

How to use reference angles to evaluate trigonometric functions

The definition of a **Reference Angle** states that:

How to you find a reference angle in each of the following quadrants:

II: III: IV:

III. Trigonometric Functions of Real Numbers

To find the value of a trigonometric function of any angle θ, you:

1)

2)

3)

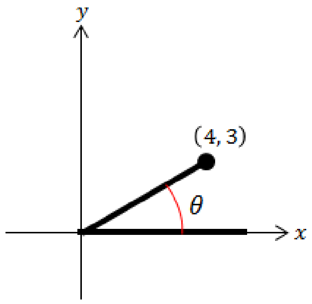
What you should learn:

How to evaluate trigonometric functions of real numbers

Section 4.4 Examples – Trigonometric Functions of Any Angle

(1) Determine the exact values of the six trigonometric functions of the angle θ.

a)



b) $\sin\theta = \frac{3}{5}$, θ lies in Quadrant II

(2) Find the reference angle θ' for the special angle θ.

$$\theta = 120°$$

(3) Find the exact value for each function for the given angle for $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$.

$$\theta = 30°$$

a) $(f + g)(\theta)$

b) $(g - f)(\theta)$

c) $[g(\theta)]^2$

d) $(fg)(\theta)$

e) $f(2\theta)$

f) $g(-\theta)$

Section 4.5 Graphs of Sine and Cosine Functions

Objective: In this lesson you learned how to sketch the graph of sine and cosine functions and translations of these functions.

Important Vocabulary			
Sine Curve	**One Cycle**	**Amplitude**	**Phase Shift**

I. Basic Sine and Cosine Curves

What you should learn:

How to sketch the graphs of basic sine and cosine functions

For $0 \leq x \leq 2\pi$, the sine function has its maximum point at

_______________________________, its minimum point at

_______________________________, and its intercepts at

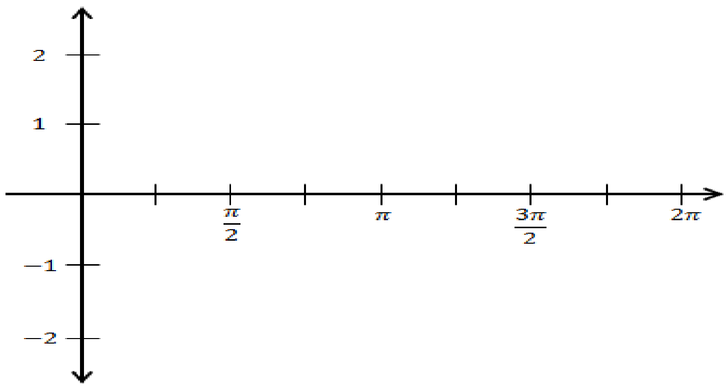
_______________________________.

For $0 \leq x < 2\pi$, the cosine function has its maximum point(s) at _________________________, its

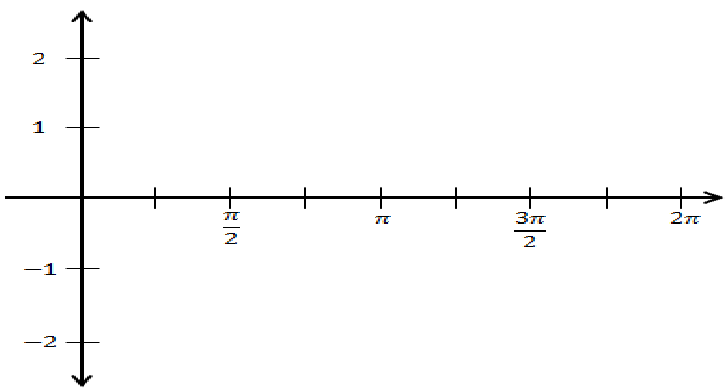
minimum point at _______________________________, and its intercepts at

_______________________________.

Sketch the sine curve on the interval $[0, 2\pi]$



Sketch the cosine curve on the interval $[0, 2\pi]$



II. Amplitude and Period of Sine and Cosine Curves

The constant factor a in $y = a \sin x$ acts as:

What you should learn:

How to use amplitude and period to help sketch the graphs of sine and cosine functions

If $|a| > 1$, the basic sine curve is ______________________. If $|a| < 1$, the basic sine curve is

______________________. The result is that the graph of $y = a \sin x$ ranges between

______________________ instead of between -1 and 1. The absolute value of a is the

______________________ of the function $y = \text{a}\sin x$.

The graph of $y = -0.5 \sin x$ is a(n) _____________________________ in the x-axis of the graph of

$y = 0.5 \sin x$.

Let b be a positive real number. The **period** of $y = a \sin bx$ and $y = a \cos bx$ is __________. If

$0 < b < 1$, the period of $y = a \sin bx$ is ________________ than 2π represents a

_____________________________ of the graph of $y = a \sin bx$. If $b > 1$, the period of $y = a \sin bx$

is ___________ than 2π represents a _____________________________ of the graph of $y = \text{a}\sin bx$.

III. Translations of Sine and Cosine Curves

The constant c in the general equations $y = a \sin(bx - c)$ and

$y = a \cos(bx - c)$ creates:

What you should learn:

How to sketch translations of graphs of sine and cosine functions

Comparing $y = a \sin bx$ with $y = a \sin(bx - c)$, the graph of $y = a \sin(bx - c)$ completes one

cycle from ___________________ to ___________________. By solving for x, you can find the interval

for one cycle is found to be ________ to ________________. This implies that the period of

$y = a \sin(bx - c)$ is ________ and the graph of $y = a \sin(bx - c)$ is the graph of $y = a \sin bx$

sifted by the amount ________.

The constant d in the equation $y = d + a \sin(bx - c)$ causes a(n)

_____________________________. For $d > 0$, the shift is _____________________________.

For $d < 0$, the shift is _____________________________. The graph oscillates about

_____________________________.

Section 4.5 Examples – Graphs of Sine and Cosine Functions

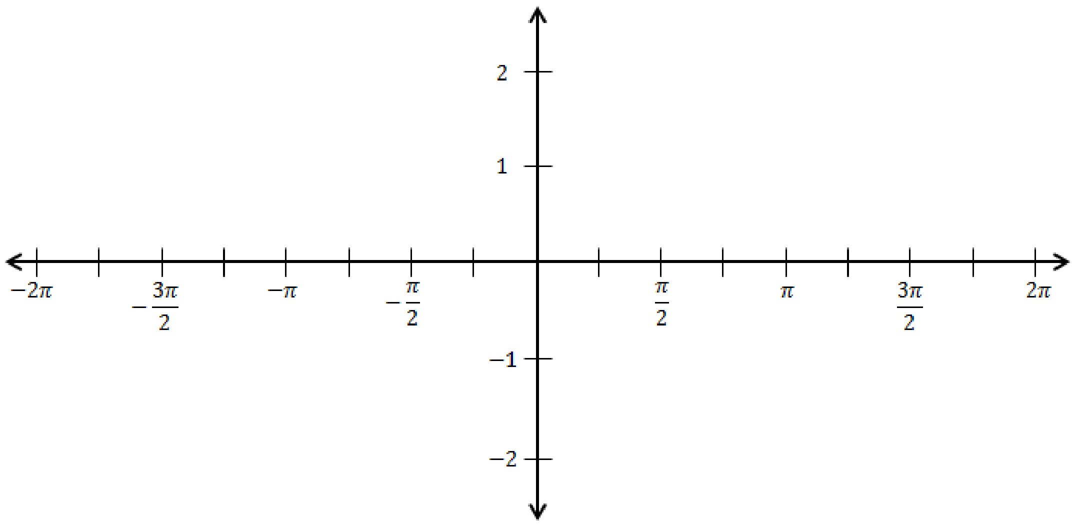
(1) Describe the translations occurring from the graph of f to the graph of g.

a) $f(x) = \sin x$
$g(x) = \sin(x - \pi)$

b) $f(x) = \cos x$
$g(x) = -\cos x$

(2) Sketch 2 full periods of the graphs of f and g on the same axes.

$f(x) = \sin x$
$g(x) = -\sin\left(x + \frac{\pi}{2}\right)$



Section 4.6 Graphs of Other Trigonometric Functions

Objective: In this lesson you learned how to sketch the graphs of other trigonometric functions.

I. Graph of the Tangent Function

What you should learn:

How to sketch the graphs of tangent functions

Because the tangent function is odd, the graph of

$y = \tan x$is symmetric with respect to the ________________.

The period of the tangent function is _______. The tangent

function has vertical asymptotes at $x =$ _____________, where n is an integer. The domain of the

tangent function is ____________________________, and the range of the function is (_____,

_____).

Describe how to sketch the graph of a function of the form $y = a\tan(bx - c)$.

1)

2)

3)

4)

II. Graph of the Cotangent Function

What you should learn:

How to sketch the graphs of cotangent functions

The period of the cotangent function is _______. The domain of

the cotangent function is __________________________, and

the range of the cotangent function is (_____, _____).

The vertical asymptotes of the cotangent function occur at $x =$ __________, where n is an integer.

III. Graphs of the Reciprocal Functions

What you should learn:

How to sketch the graphs of secant and cosecant functions

At a given value of x, the y-coordinate of $\csc x$ is the reciprocal

of the y-cooridnate of ______________.

The graph of $y = \csc x$ is symmetric with respect to the

_______________. The period of the cosecant function is ________. The cosecant function has

vertical asymptotes at $x =$ ________, where n is an integer. The domain of the cosecant function is

____________________________, and the range of the cosecant functions is

____________________________.

At a given value of x, the y-coordinate of $\sec x$ is the reciprocal of the y-coordinate of

_______________. The graph of $y = \sec x$ is symmetric with respect to the _______________. The period of the secant function is _______________. The secant function has vertical asymptotes at $x =$ _______________. The domain of the secant function is _____________________________, and the range of the secant function is _____________________________.

To sketch a graph of a secant or cosecant function, you:

1)

2)

3)

4)

In comparing the graphs of cosecant and secant functions with those of the sine and cosine functions, note that the “hills” and “valleys” are _____________________________.

Section 4.6 Examples – Graphs of Other Trigonometric Functions

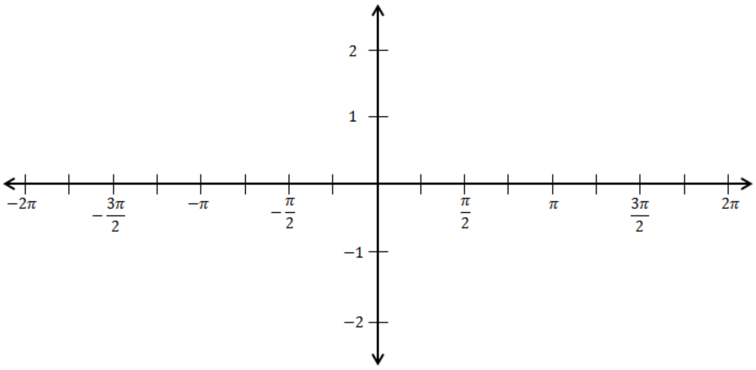
(1) Describe the translations occurring from the graph of f to the graph of g.

$f(x) = \tan x$

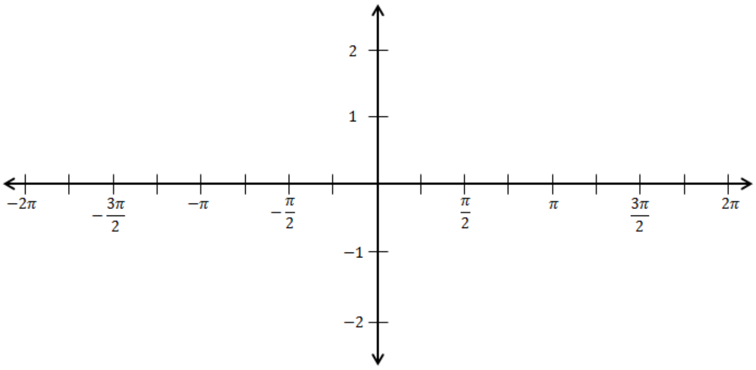
$g(x) = \tan\left(x + \frac{\pi}{4}\right)$

(2) Sketch 2 full periods of the graphs of f

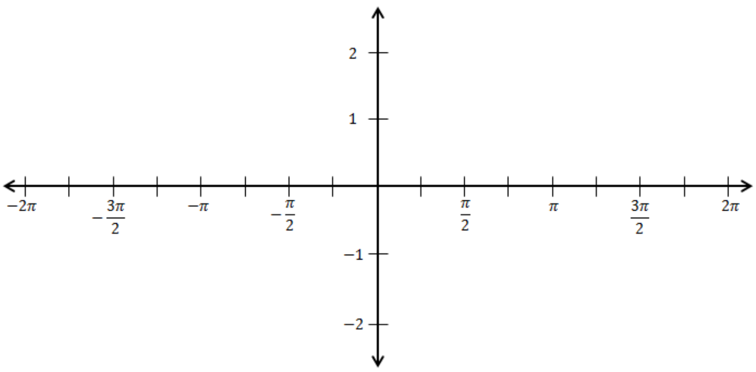
a. $f(x) = \frac{1}{2}\tan x$



b. $f(x) = \csc\frac{x}{2}$



c. $f(x) = -\frac{1}{2}\sec x$



Section 4.7 Inverse Trigonometric Functions

Objective: In this lesson you learned how to evaluate the inverse trigonometric functions and how to evaluate the composition of trigonometric functions.

Important Vocabulary		
Inverse Sine Function	**Inverse Cosine Function**	**Inverse Tangent Function**

I. Inverse Sine Function

The **inverse sine function** is defined by:

What you should learn:

How to evaluate inverse sine functions

The domain of $y = \arcsin x$ is [_____, _____]. The range of $y = \arcsin x$ is [_____, _____].

II. Other Inverse Trigonometric Functions

The **inverse cosine function** is defined by:

What you should learn:

How to evaluate other inverse trigonometric functions

The domain of $y = \arccos x$ is [_____, _____]. The range of $y = \arccos x$ is [_____, _____].

The **inverse tangent function** is defined by:

The domain of $y = \arctan x$ is (_____, _____). The range of $y = \arctan x$ is (_____, _____).

III. Compositions of Functions

State the Inverse Property for the Sine function.

What you should learn:

How to evaluate compositions of trigonometric functions

State the Inverse Property for the Cosine function.

State the Inverse Property for the Tangent function.

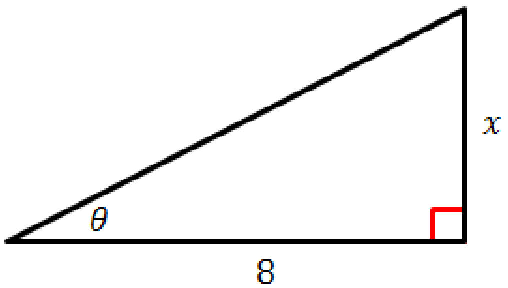
Section 4.7 Examples – Inverse Trigonometric Functions

(1) Use a calculator to approximate the value of the expression in radians and degrees.

a) $\arcsin 0.45$

b) $\cos^{-1} 0.28$

(2) Use an inverse trigonometric function to write θ as a function of x.



Section 4.8 Applications and Models

Objective: In this lesson you learned how to use trigonometric functions to model and solve real-life problems.

Important Vocabulary
Bearing

I. Trigonometry and Bearings

What you should learn:

How to solve real-life problems involving directional bearings

Used to give directions in surveying and navigation, a **bearing** measures:

The bearing $N\ 70°\ E$ means:

II. Harmonic Motion

What you should learn:

How to solve real-life problems involving harmonic motion

The vibration, oscillation, or rotation of an object under ideal conditions such that the object's uniform and regular motion can be described by a sine or cosine function is called

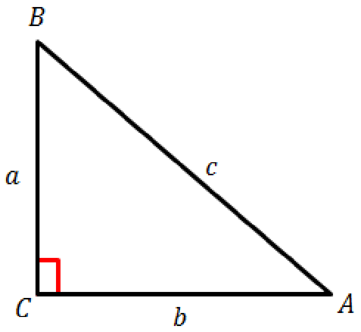
_____________________________.

A point that moves on a coordinate line is said to be in **simple harmonic motion** if:

The simple harmonic motion has amplitude ________, period ________, and frequency ________.

Section 4.8 Examples – Applications and Models

(1) Solve the right triangle shown in the figure.
$A = 30°, b = 10$



(2) A ship leaves port at noon and has a bearing of $S\ 29°\ W$. The ship sails at 20 knots. How many nautical miles south and how many nautical miles west does the ship travel by 6:00 P.M.?