Chapter 3 – Exponential and Logarithmic Functions

Vocabulary

Exponential function	Natural Base *e*
Common Logarithmic Function	Natural Logarithmic Function
Change-of-base formula	

Section 3.1 Exponential Functions and Their Graphs

Objective: In this lesson you learned how to recognize, evaluate, and graph exponential functions.

Important Vocabulary	
Exponential Function	**Natural Base e**

I. Exponential Functions

> What you should learn:
>
> How to recognize and evaluate exponential functions with base a

Polynomial functions and rational functions are examples of

_____________________________ functions.

The **exponential function f with base a** is denoted by

_____________________________, where $a \geq 0, a \neq 1$, and x is any real number.

II. Graphs of Exponential Functions

> What you should learn:
>
> How to graph exponential functions with base a

For $a > 1$, is the graph of $f(x) = a^x$ increasing or decreasing

over its domain? _____________________________

For $a > 1$, is the graph of $g(x) = a^{-x}$ increasing or decreasing

over its domain? _____________________________

For the graph of $y = a^x$ or $y = a^{-x}$, $a > 1$, the domain is _________________, the range is

_________________, and the y-intercept is _________________. Also, both graphs have

________________________ as a horizontal asymptote.

III. The Natural Base e

> What you should learn:
>
> How to recognize, evaluate, and graph exponential functions with base e

The **natural exponential function** is given by the function

_________________.

For the graph of $f(x) = e^x$, the domain is

_________________,the range is _________________,and the y-intercept is _________________.

The number e can be approximated by the expression _________________ for large values of x.

IV. Applications

After t years, the balance A in an account with principal P and annual interest rate r (in decimal form) is given by the formulas:

What you should learn:
How to recognize, evaluate, and graph exponential functions with base e

For n compounding's per year: _______________________

For continuous compounding: _______________________

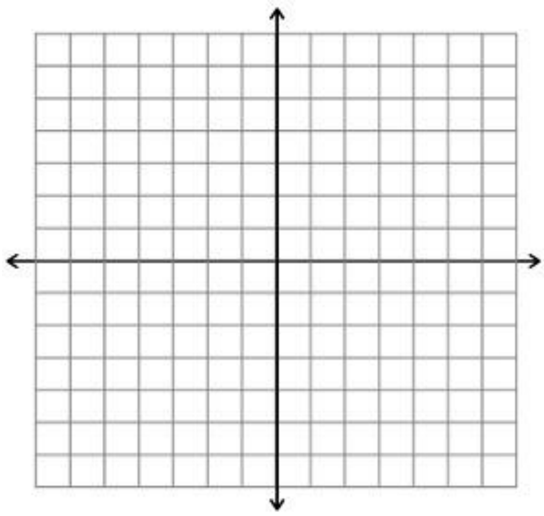
Section 3.1 Examples – Exponential Functions and Their Graphs

(3) Using what you know from Chapter 1 (horizontal/vertical shifts, reflections, etc), describe the transformation from the graph of f to the graph of g.

$$f(x) = 3^x \qquad g(x) = 3^{x-5}$$

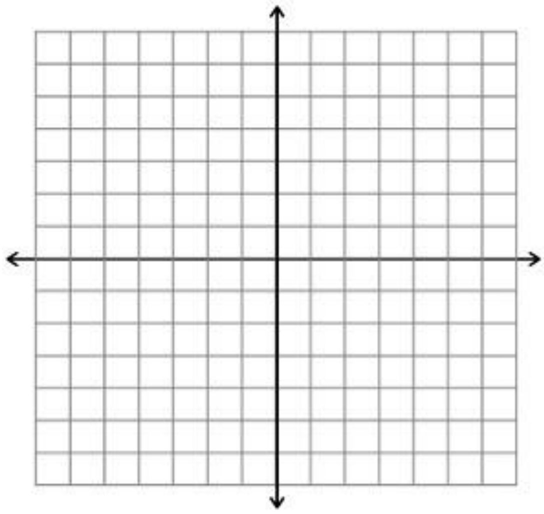
(4) Sketch a graph of the function by finding the asymptote(s) and calculating a few other points. State the domain and range in interval notation.

$$f(x) = 3^{x-1}$$



(5) Sketch a graph of the function by finding the asymptotes and calculating a few other points. State the domain and range in interval notation.

$$f(x) = 2 + e^{x-2}$$



Section 3.2 Logarithmic Functions and Their Graphs

Objective: In this lesson you learned how to recognize, evaluate, and graph logarithmic functions.

Important Vocabulary	
Common Logarithmic Function	**Natural Logarithmic Function**

I. Logarithmic Functions

What you should learn:

How to recognize and evaluate logarithmic functions with base a

The logarithmic function with base a is the

______________________________ of the exponential function

$f(x) = a^x$.

The **logarithmic function with base a** is defined as ________________________, for $x > 0, a > 0$,

and $a \neq 1$, if and only if $x = a^y$. The notation " $\log_a x$ " is read as

"__________________________________."

The equation $x = a^y$ in exponential form is equivalent to the equation _____________________ in

logarithmic form.

When evaluating logarithms, remember that a logarithm is a(n) _____________________. This means

that $\log_a x$ is the _____________________ to which a must be raised to obtain _______.

Complete the following logarithm properties:

1) $\log_a 1 =$ ____________

2) $\log_a a =$ ____________

3) $\log_a a^x =$ ___________

4) $a^{\log_a x} =$ ____________

5) If $\log_a x = \log_a y$, then ___________

II. Graphs of Logarithmic Functions

What you should learn:

How to graph logarithmic functions with base a

For $a > 1$, is the graph of $f(x) = \log_a x$ increasing or decreasing over its domain?

____

For the graph of $f(x) = \log_a x, a > 1$, the domain is _________________, the range is _________________, and the x-intercept is _________________.

Also, the graph has ___________________________ as a vertical asymptote. The graph of $f(x) = \log_a x$ is a reflection of the graph of $f(x) = a^x$ over the line ___________.

III. The Natural Logarithmic Function

What you should learn:

How to recognize, evaluate, and graph natural logarithmic functions

Complete the following natural logarithm properties:

1) $\ln 1 =$ ____________
2) $\ln e =$ ____________
3) $\ln e^x =$ ____________
4) $e^{\ln x} =$ ____________
5) If $\ln x = \ln y$, then ____________.

Section 3.2 Examples – Logarithmic Functions and Their Graphs

(1) Write the logarithmic equation in exponential form.

(a) $\log_4 64 = 3$

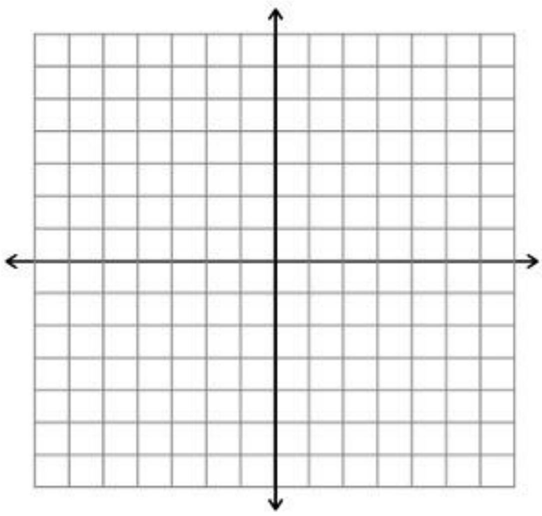
(b) $\log_5 \sqrt[3]{25} = \frac{2}{3}$

(2) Using what you know from Chapter 1 (horizontal/vertical shifts, reflections, etc), describe the transformation from the graph of f to the graph of g.

$$f(x) = \log_2 x \qquad g(x) = -2 + \log_2(x+3)$$

(3) Sketch a graph of the function by finding the asymptote(s) and calculating a few other points. State the domain and range in interval notation.

$$f(x) = \ln(x+1)$$



Section 3.3 Properties of Logarithms

Objective: In this lesson you learned how to rewrite logarithmic functions with different bases and how to use properties of logarithms to evaluate, rewrite, expand, or condense logarithmic expressions.

Important Vocabulary
Change-of-Base Formula

I. Change of Base

Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. The **change-of-base formula** states that:

> What you should learn:
>
> How to rewrite logarithms with different bases

Explain how to use a calculator to evaluate $\log_8 20$.

II. Properties of Logarithms

Let a be a positive number such that $a \neq 1$; let n be a real number; and let u and v be positive real numbers.

> What you should learn:
>
> How to use properties of logarithms to evaluate or rewrite logarithmic expressions

Complete the following logarithm properties:

1) $\log_a(uv) =$ ______________________

2) $\log_a \frac{u}{v} =$ ______________________

3) $\log_a u^n =$ ______________________

III. Rewriting Logarithmic Expressions

To expand a logarithmic expression means to:

> What you should learn:
>
> How to use properties of logarithms to expand or condense logarithmic expressions

To condense a logarithmic expression means to:

IV. Applications of Properties of Logarithms

What you should learn:

How to use properties of logarithmic functions to model and solve real-life problems

One way of finding a model for a set of nonlinear data is to take the natural log of each of the x-values and y-values of the data set. If the points are graphed and fall on a straight line, then the x-values and y-values are related by the equation ______________________________, where m is the slope of the straight line.

Section 3.3 Examples – Properties of Logarithms

(1) Rewrite the logarithm as a ratio of (a) common logarithms and (b) natural logarithms.

$$\log_5 x$$

(2) Use the properties of logarithms to rewrite and simplify the logarithmic expression.

$$\log_2 4^2 \cdot 3^4$$

(3) Use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms.

$$\ln \frac{xy}{z}$$

(4) Condense the expression to the logarithm of a single quantity.

$$3 \log x + 2 \log y - 4 \log z$$

Section 3.4 Solving Exponential and Logarithmic Equations

Objective In the lesson you learned how to solve exponential and logarithmic equations.

I. Introduction

What you should learn:
How to solve simple exponential and logarithmic equations

State the One-to-One Property for exponential equations.

State the One-to-One Property for logarithmic equations.

State the Inverse Property for exponential equations and for logarithmic equations.

Describe some strategies for using the One-to-One Properties and the Inverse Properties to solve exponential and logarithmic equations.

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II. Solving Exponential Equations

What you should learn:
How to solve more complicated exponential equations

Describe how to solve the exponential equation $10^x = 90$ algebraically.

III. Solving Logarithmic Equations

Describe how to solve the logarithmic equation $\log_6(4x-7) = \log_6(8-x)$ algebraically.

What you should learn:

How to solve more complicated logarithmic equations

IV. Applications of Solving Exponential and Logarithmic Equations

Use the formula for continuous compounding $A = Pe^{rt}$, to find out how long it will take $\$1500$ to triple in value if it is invested at 12% interest, compounded continuously.

What you should learn:

How to use exponential and logarithmic equations to model and solve real-life problems

Section 3.4 Examples – Solving Exponential and Logarithmic Equations

(1) Solve the exponential equation.

$$5^x = \frac{1}{625}$$

(2) Solve the logarithmic equation.

$$\ln(2x - 1) = 5$$

(3) Solve the equation. Round your answer to three decimal places.

(a) $7 - 2e^x = 5$ (b) $\log x^2 = 6$

Section 3.5 Exponential and Logarithmic Models

Objective: In this lesson you learned how to use exponential growth models, exponential decay models, logistic models, and logarithmic models to solve real-life problems.

I. Introduction

What you should learn:

How to recognize the five most common types of models involving exponential or logarithmic functions

The **exponential growth model** is ____________________.

The **exponential decay model** is ____________________.

The **Gaussian model** is ____________________.

The **logistic growth model** is ____________________.

Logarithmic models are ____________________.and ____________________.

II. Exponential Growth and Decay

What you should learn:

How to use exponential growth and decay functions to model and solve real-life problems

To estimate the age of dead organic matter, scientists use the carbon dating model ____________________________, which denotes the ratio R of carbon 14 to carbon 12 present at any time t (in years).

III. Gaussian Models

What you should learn:

How to use Gaussian functions to model and solve real-life problems

The Gaussian model is commonly used in probability and statistics to represent populations that are

____.

On a bell-shaped curve, the average value for a population is where the

_______________________________ of the function occurs.

IV. Logarithmic Models

What you should learn:

How to use logarithmic functions to model and solve real-life problems

The number of kitchen widgets y (in millions) demanded each year is given by the model $y = 2 + 3\ln(x + 1)$, where $x = 0$ represents the year 2000 and $x \geq 0$. Find the year in which the number of kitchen widgets demanded will be 8.6 million.