Chapter 2 – Polynomial and Rational Functions

Vocabulary

Linear Function	Quadratic Function
Standard form (of a quadratic)	Axis of Symmetry
Vertex	Continuous
The Leading Coefficient Test	Extrema
Relative minimum	Relative Maximum
Repeated zero	Multiplicity
Intermediate Value Theorem	Synthetic division
Remainder Theorem	Factor Theorem
Rational Zero Test	Imaginary unit
Complex number	Complex conjugate
The Fundamental Theorem of Algebra	Linear Factorization Theorem
Conjugates	Rational function
Vertical asymptote	Horizontal asymptote
Slant asymptote	

Section 2.1 Quadratic Functions

Objective: In this lesson you learned how to sketch and analyze graphs of quadratic functions

Important Vocabulary		
Constant Function	**Linear Function**	**Quadratic Function**
Axis of Symmetry	**Vertex**	

I. The Graph of a Quadratic Function

What you should learn:

How to analyze graphs of quadratic functions

Let n be a nonnegative integer and let $a_n, a_{n-1}, \dots, a_2, a_1, a_0$ be real numbers with $a_n \neq 0$. A **polynomial function of x with degree n** is given by:

A **quadratic function** is a polynomial function of ______________________ degree. The graph of

a quadratic function is a special "U-shaped" curved called a(n) ________________________.

If the leading coefficient of a quadratic function is positive, the graph of the function opens

____________________ and the vertex of the parabola is the ____________________ point on the

graph. If the leading coefficient of a quadratic function is negative, the graph of the function opens

________________________ and the vertex of the parabola is the ________________________ point

on the graph.

II. The Standard Form of a Quadratic Function

What you should learn:

How to write quadratic functions in standard form and use the results to sketch the graphs of functions

The **standard form of a quadratic function is**

____.

For a quadratic function in standard form, the axis (of

symmetry) of the associated parabola is _____________ and the vertex is (_____, _____).

To write a quadratic function in standard form:

To find the x-intercepts of the graph of $f(x) = ax^2 + bx + c$:

III. Finding Minimum and Maximum Values

What you should learn:

How to find minimum and maximum values of quadratic functions in real-life applications

For a quadratic function in the form $f(x) = ax^2 + bx + c$,

when $a > 0$, f has a minimum that occurs at ____________.

When $a < 0$, f has a maximum that occurs at ____________.

To find the minimum or maximum value:

Section 2.1 Examples – Quadratic Functions

(1) Describe the graph of $f(x) = 2x^2 + 8x + 8$ by identifying the vertex, axis of symmetry, x-intercepts, and direction the graph opens.

(2) Write the standard form of the equation of the parabola whose vertex is $(1, 2)$ and that passes through the point $(3, -6)$.

Section 2.2 Polynomial Functions of Higher Degree

Objective: In this lesson you learned how to sketch and analyze graphs of polynomial functions

Important Vocabulary

Continuous **Extrema** **Relative minimum** **Relative maximum**

I. Graphs of Polynomial Functions

What you should learn:

How to use transformations to sketch graphs of polynomial functions

Name two basic features of the graphs of polynomial functions.

1)

2)

Will the graph of $g(x) = x^7$ look more like the graph of $f(x) = x^2$ or the graph of $f(x) = x^3$? Explain.

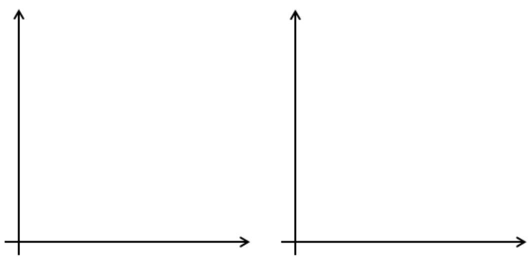
II. The Leading Coefficient Test

What you should learn:

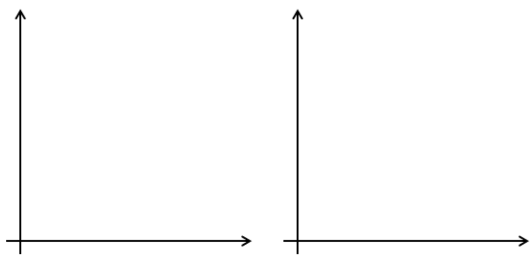
How to use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions

State the **Leading Coefficient Test**:

1) When n is odd



2) When n is even



III. Zeros of Polynomials Functions

What you should learn:

How to find and use zeros of polynomial functions as sketching aids

Let f be a polynomial function of degree n. The function f has at most ______ real zeros. The graph of f has at most ____________ **relative extrema**.

Let f be a polynomial function and let a be a real number. List four equivalent statements about the real zeros of f.

1)

2)

3)

4)

For a polynomial function, a factor of $(x-a)^k, k>1$, yields a **repeated zero**:

1)

2)

If a polynomial function f has a repeated zero $x=3$ with multiplicity 4, the graph of f ___________________ the x-axis at $x=$ ______. If f has a repeated zero $x=4$ with multiplicity 3, the graph of f ___________________ the x-axis at $x=$ ______.

IV. The Intermediate Value Theorem

What you should learn:

How to use the Intermediate Value Theorem to help locate zeros of polynomial functions

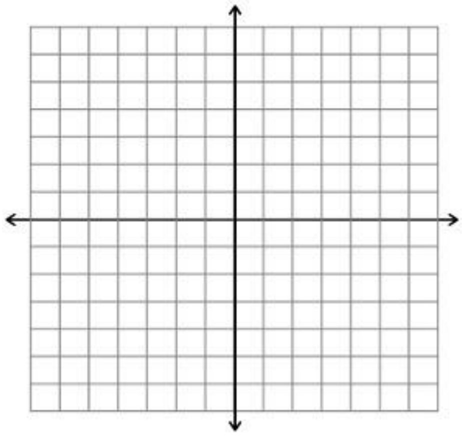
State the Intermediate Value Theorem:

Describe how the Intermediate Value Theorem can help in locating the real zeros of a polynomial function f.

Section 2.2 Examples – Polynomial Functions of Higher Degree

(1) Use the Leading Coefficient Test to determine the right and left hand behavior of the graph. Then sketch what the rest of the graph might look like.

$f(x) = -x^3 + 2x - 1$



(2) Find all the real zeros of the polynomial function.

$$h(t) = t^2 - 6t + 9$$

(3) Find a polynomial function with the given zeros, multiplicities, and degree.

Zero: -1, multiplicity: 2

Zero: -2, multiplicity: 1

Degree: 3

Section 2.3 Real Zeros of Polynomial Functions

Objective: In this lesson you learned how to use long division and synthetic division to divide polynomials by other polynomials and how to find the rational and real zeros of polynomial functions

Important Vocabulary			
Long division of polynomials	**Division Algorithm**	**Synthetic Division**	
Remainder Theorem	**Factor Theorem**	**Upper bound**	
Lower bound	**Improper**	**Proper**	**Rational Zero Test**

I. Long Division of Polynomials

What you should learn:

How to use long division to divide polynomials by other polynomials

When dividing a polynomial $f(x)$ by another polynomial $d(x)$, if the remainder $r(x) = 0$, $d(x)$ ________________________ into $f(x)$.

If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x)q(x) + r(x)$$

where:

The rational expression $f(x)/d(x)$ is **improper** if:

The rational expression $r(x)/d(x)$ is **proper** if:

Before applying the Division Algorithm, you should:

II. Synthetic Division

What you should learn:

How to use synthetic division to divide polynomials by binomials of the form $(x - k)$

Can synthetic division be used to divide a polynomial by $x^2 - 5$? Explain.

Can synthetic division be used to divide a polynomial by $x + 4$? Explain.

To divide $ax^3 + bx^2 + cx + d$ by $x - k$, use the following pattern:

III. The Remainder and Factor Theorems

What you should learn:

How to use the Remainder and Factor Theroems

To use the Remainder Theorem to evaluate a polynomial function $f(x)$ at $x = k$, you:

To use the Factor Theorem to show that $(x - k)$ is a factor of a polynomial function $f(x)$, you:

List three facts about the remainder r, obtained in the synthetic division of $f(x)$ by $x - k$:

1)

2)

3)

IV. The Rational Zero Test

What you should learn:

How to use the Rational Zero Test to determine possible rational zeros of polynomial functions

Describe the purpose of the Rational Zero Test:

State the **Rational Zero Test**:

To use the Rational Zero Test, you:

V. Other Tests for Zeros of Polynomials

What you should learn:

How to use Descartes's Rule of Signs and the Upper and Lower Bound Rules to find zeros of polynomials

State Descartes's Rule of Signs:

Section 2.3 Examples – Real Zeros of Polynomial Functions

(1) Use long division to divide.

$$(x + 8 + 6x^3 + 10x^2) \div (2x^2 + 1)$$

(2) Use synthetic division to divide.

$$(3x^3 - 17x^2 + 15x - 25) \div (x - 5)$$

(3) Use the Remainder Theorem and Synthetic Division to evaluate the function at each given value.

$$g(x) = 2x^6 + 3x^4 - x^2 + 3$$

a) $g(2)$ b) $g(1)$ c) $g(3)$ d) $g(-1)$

(4) Find **ALL** real zeros of the polynomial function.

$f(x) = 4x^3 + 7x^2 - 11x - 18$

Section 2.4 Complex Numbers

Objective: In this lesson you learned how to perform operations with complex numbers

Important Vocabulary		
Complex Number	**Complex conjugates**	**Imaginary axis**
Real axis	**Bounded**	**Unbounded**

I. The Imaginary Unit i

What you should learn:

How to use the imaginary unit i to write complex numbers

Mathematicians created an expanded system of numbers using the **imaginary unit i**, defined as $i =$ ________, because:

By definition, $i^2 =$ ______.

For the **complex number** $a + bi$, if $b = 0$, the number $a + bi = a$ is a(n) __________________________. If $b \neq 0$, the number $a + bi$ is a(n) ___________________________.

If $a = 0$, the number $a + bi = bi$, where $b \neq 0$, is called a(n) ____________________________________. The set of complex numbers consists of the set of ___________________________ and the set of ____________________________________.

Two complex numbers $a + bi$ and $c + di$, written in standard form, are equal to each other if:

II. Operations with Complex Numbers

What you should learn:

How to add, subtract, and multiply complex numbers

To add two complex numbers, you:

To subtract two complex numbers, you:

The additive identity in the complex number system is _____________.

The additive inverse of the complex number $a + bi$ is ________________.

To multiply two complex numbers $a + bi$ and $c + di$, you:

III. Complex Conjugates

What you should learn:

How to use complex conjugates to write the quotient of two complex numbers in standard form

The product of a pair of complex conjugates is a(n)

___________ number.

To find the quotient of the complex numbers $a + bi$ and $c + di$,

where c and d are not both zero, you:

Section 2.4 Examples – Complex Numbers

(1) Perform the indicated operation. Write the answer in standard form.

a) $(3 - i) + (2 + 3i)$

b) $3 - (-2 + 3i) + (-5 + i)$

c) $(2 - i)(4 + 3i)$

d) $\frac{2+3i}{4-2i}$

(2) Solve the quadratic equation.

a) $x^2 + 4 = 0$

b) $3x^2 - 2x + 5 = 0$

Section 2.5 The Fundamental Theorem of Algebra

Objective: In this lesson you learned how to determine the numbers of zeros of polynomial functions and find them

Important Vocabulary		
Fundamental Theorem of Algebra	**Linear Factorization Theorem**	**Conjugates**

I. The Fundamental Theorem of Algebra

In the complex numbers system, every nth-degree polynomial function has ___________________________ zeros. An nth-degree polynomial can be factored into ___________________________ linear factors.

What you should learn:

How to use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function and find all zeros of polynomial functions, including complex zeros

II. Conjugate Pairs

Let $f(x)$ be a polynomial function that has real coefficients. If $a + bi$, where $b \neq 0$, is a zero of the function, then we know that ___________________________ is also a zero of the function.

What you should learn:

How to find conjugate pairs of complex zeros

III. Factoring a Polynomial

To write a polynomial of degree $n > 0$ with real coefficients as a product without complex factors, write the polynomial as:

What you should learn:

How to find zeros of polynomials by factoring

A quadratic factor with no real zeros is said to be __________________.

Section 2.5 Examples – The Fundamental Theorem of Algebra

(1) For the following problem, (a) find **ALL** the zeros of the function, and (b) write the polynomial as a product of linear factors. You might have to use the **Quadratic Formula** if you can't factor.

$$f(y) = 81y^4 - 625$$

(2) Find a fourth degree polynomial that has $2, 2,$ and $4 - i$ as zeros.

Section 2.6 Rational Functions and Asymptotes

Objective: In this lesson you learned how to determine the domains and find asymptotes of rational functions

Important Vocabulary		
Rational function	**Vertical asymptotes**	**Horizontal asymptotes**

I. Introduction to Rational Functions

What you should learn:

How to find the domains of rational functions

The domain of a rational function of x incudes all real numbers except:

To find the domain of a rational function of x, you:

II. Horizontal and Vertical Asymptotes

What you should learn:

How to find horizontal and vertical asymptotes of graphs of rational functions

The notation "$f(x) \to 5$ as $x \to \infty$" means:

Let f be the rational function given by

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0}$$

where $N(x)$ and $D(x)$ have no common factors.

1) The graph of f has vertical asymptotes at ____.

2) The graph of f has at most one horizontal asymptote determined by

____.

   a. If $n < m$, ____

   b. If $n = m$, ____

   c. If $n > m$, ____

What makes a hole in the graph of a rational function?

Section 2.7 Graphs of Rational Functions

Objective: In this lesson you learned how to sketch graphs of rational functions

Important Vocabulary

Slant (or oblique) asymptote

I. The Graph of a Rational Function

What you should learn:

How to analyze and sketch graphs of rational functions

To sketch the graph of the rational function

$f(x) = N(x)/D(x)$, where $N(x)$ and $D(x)$ are polynomials

with no common factors, you:

1.

2.

3.

4.

5.

6.

7.

II. Slant Asymptotes

What you should learn:

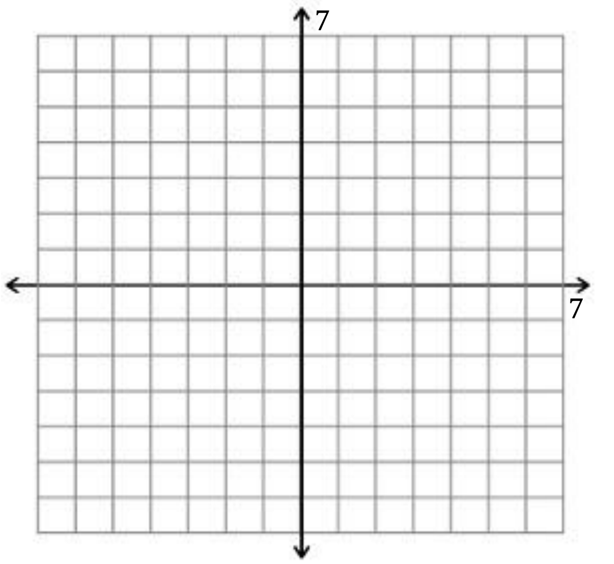
How to sketch graphs of rational functions that have slant asymptotes

To find the equation of a slant asymptote, you:

Section 2.6/2.7 Examples – Rational Functions

1) Graph the function (a) by finding the intercepts, (b) finding the asymptotes, and (c) using an xy-chart to fill in the rest.

   $f(x) = -\frac{2}{x+3}$



2) Graph the function (a) by finding the intercepts, (b) finding the asymptotes, and (c) using an xy-chart to fill in the rest.

   $f(x) = \frac{2x^2-5x+5}{x-2}$