Chapter 1 – Functions and Their Graphs

Vocabulary

Slope	Parallel	Perpendicular
Point-slope form	Slope-intercept form	Horizontal line
Vertical line	General form (of a line)	Linear function
Domain	Range	Set-Builder Notation
Interval Notation	Function	Independent Variable
Dependent Variable	Relation	Function notation
Vertical Line Test	Increasing	Decreasing
Relative minimum	Relative maximum	Relative extrema
Even Function	Odd Function	Difference quotient
Parent function	Vertical Shift	Horizontal Shift
Reflection	Rigid transformation	
Non-rigid Transformation	Combinations of functions	
Function Composition	Inverse Function	One-to-one
Horizontal Line Test		

Section 1.1 Lines in the Plane

Objective: In this lesson you will review how to find and use the slope of a line to write and graph linear equations

Important Vocabulary			
Slope	**Parallel**	**Perpendicular**	**Point-Slop Form**
Slope-Intercept Form	**General Form**	**Horizontal Line**	**Vertical Line**
Linear Function			

I. The Slope of a Line

What you should learn:

How to find the slopes of lines

The formula for the **slope** of a line passing through the points (x_1, y_1) and (x_2, y_2) is $m =$ ___________________.

A line whose slope is positive ___________________ from left to right.

A line whose slope is negative ___________________ from left to right.

A line with zero slope is ___________________________.

A line with undefined slope is ___________________________.

II. The Point-Slope Form of the Equation of a Line

What you should learn:

How to write linear equations given points on lines and their slopes

The **point-slope** form of the equation of a line is

____

This form of equation is best used to find the equation of a line when:

The **two-point form** of the equation of a line is ____.

The two-point form of a line is best used to find the equation of a line when:

A **linear function** has the form __________________________. Its graph is a __________________

that has slope ______ and a y-intercept at (_____, _____).

III. Sketching Graphs of Lines

What you should learn:

How to use slope-intercept forms of linear equations to sketch lines.

The **slope-intercept form** of the equation of a line is

__________________________, where m is the

___________________ and the y-intercept is (_____, _____).

The equation of a **horizontal line** is ______________. The slope of a horizontal line is ________. The y-coordinate of every point on the graph of a horizontal line is ________.

The equation of a **vertical line** is ______________. The slope of a vertical line is __________________________. The x-coordinate of every point on the graph of a vertical line is ________.

The **general form** of the equation of a line is __________________________.

Every line has an equation that can be written in ____.

IV. Parallel and Perpendicular Lines

What you should learn:

How to use slope to identify parallel and perpendicular lines

The relationship between the slope of two lines that are parallel is:

The relationship between the slope of two lines that are perpendicular is:

A line that is parallel to a line whose slope is 2 has a slope of ________.

A line that is perpendicular to a line whose slope is 2 has a slope of ________.

Section 1.1 Examples – Lines in the Plane

(1) Find the point-slope form of the equation that passes through the given point and has the indicated slope.

$(-3, 6)$ $\qquad m = -2$

(2) Decide whether the two lines are *parallel*, *perpendicular*, or *neither*.

$y = 4x - 1$ $\qquad 2x + 8y = 12$

(3) Determine the slope and y-intercept (if possible) of the linear equation.

$3x + 4y = 1$

Section 1.1.5 Domain and Range

Objective: In this lesson you learned how to identify domain, range, and how to write domain and range in Roster notation, Set-Builder notation, and Interval notation.

Important Vocabulary		
Domain	**Range**	**Roster Notation**
Set-Builder Notation	**Interval Notation**	

I. Domain and Range

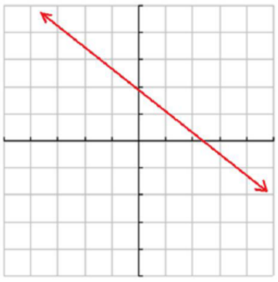
What you should learn:

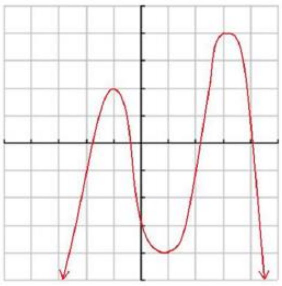
How to find (determine) domain and range from a graph and/or equation

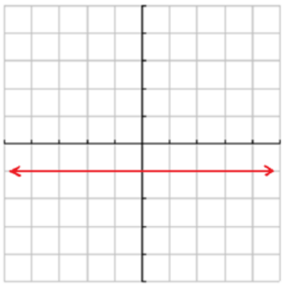
What is domain?

What is range?

How can you find (determine) domain and range when given a graph?







How can you find (determine) domain and range when given an equation?

II. Notation

A. Roster Notation –

What you should learn:

How to write domain and range in different notation formats

B. Set-Builder Notation –

C. Interval Notation –

III. Domain Restrictions

What causes domain restrictions?

What you should learn:

How to identify possible domain and range issues given a graph and/or equation

Section 1.1.5 Examples – Domain and Range

(1) Determine the domain and range of the following set of ordered pairs.

$$\{(-8,0),(6,4),(0,0),(-7,-3),(10,-5)\}$$

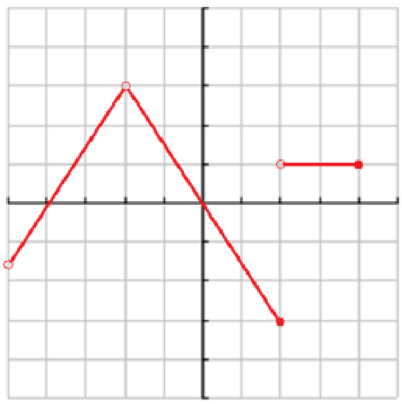
(2) Given the inequality, write its equivalent form in set-builder notation.

$$a > 12$$

(3) Given the inequality, write its equivalent form in interval notation.

$$x \leq 7$$

(4) Use the graph of the function to write the domain and range in set-builder notation and interval notation.



Section 1.2 Functions

Objective: In this lesson you learned how to evaluate functions and find their domains

Important Vocabulary				
Function	**Domain**	**Range**	**Independent Variable**	**Dependent Variable**
Relation	**Function Notation**		**Difference Quotient**	

I. Introduction to Functions

What you should learn:

How to decide whether a relation between two variables represents a function

A rule of correspondence that matches quantities from one set with items from a different set is a(n)

_______________________.

In functions that can be represented by ordered pairs, the first coordinate in each ordered pair is the __________________________ and the second coordinate is the __________________________.

Some characteristics of a function from Set A to Set B are:

1)

2)

3)

4)

To determine whether or not a relation is a function:

If any input value of a relation is matched with two or more output values:

II. Function Notation

What you should learn:

How to use function notation and evaluate functions

The symbol ________ is **function notation** for the value of f at x or simply f of x. The symbol $f(x)$ corresponds to the __________________________ for a given x.

Keep in mind that ______ is the name of the function, whereas ______ is the output value of the function at the input value x.

In function notation, the _____________is the independent variable and the _____________is the dependent variable.

A piecewise-defined function is:

III. The Domain of a Function

What you should learn:

How to find the domains of functions

The **implied domain** of a function defined by an algebraic expression is:

In general, the domain of a function excludes values that:

IV. Difference Quotients

What you should learn:

How to evaluate difference quotients

A **difference quotient** is defined as:

Section 1.2 Examples – Functions

(1) Evaluate the function at each specified value of the independent variable and simplify.

$f(a) = 3a + 5$ a) $f(-2)$ b) $f(4)$ c) $f(x + 1)$

(2) Find the domain of the function.

$s(y) = \frac{3y}{y+5}$

(3) Find the difference quotient and simplify your answer.

$g(x) = 3x - 1$ where $\frac{g(x+h)-g(x)}{h}, h \neq 0$ is the difference quotient

Section 1.3 Graphs of Functions

Objective: In this lesson you learned how to analyze the graphs of functions

Important Vocabulary		
Graph of a Function	**Greatest Integer Function**	**Step Function**
Even Function	**Odd Function**	**Interval Notation**
Vertical Line Test	**Increasing**	**Decreasing**
Relative Minimum	**Relative Maximum**	**Relative Extrema**

I. The Graph of a Function

What you should learn:

How to find the domains and ranges of functions and how to use the Vertical Line Test for functions

Explain the use of open or closed dots in the graphs of functions:

To find the domain of a function from its graph:

To find the range of a function from its graph:

The **Vertical Line Test** for functions states:

II. Increasing and Decreasing Functions

What you should learn:

How to determine intervals on which functions are increasing, decreasing, or constant

A function f is **increasing** on an interval if, for any x_1 and x_2 in the interval:

A function f is **decreasing** on an interval if, for any x_1 and x_2 in the interval:

A function f is **constant** on an interval if, for any x_1 and x_2 in the interval:

Given a graph of a function, how do you determine when a function is *increasing, decreasing*, or *constant*?

III. Relative Minimum and Maximum Values

What you should learn: How to determine relative minimum and relative maximum values of functions

A function value $f(a)$ is called a **relative minimum** of f if:

A function value $f(a)$ is called a **relative maximum** of f if:

The point at which a function changes from increasing to decreasing is a relative ___________________________. The point at which a function changes from decreasing to increasing is a relative ___________________________.

IV. Step Functions and Piecewise-Defined Functions

What you should learn:

How to identify and graph step functions and other piecewise-defined functions

Describe the graph of the greatest integer function:

Describe the graph of a piecewise-defined function:

V. Even and Odd Functions

What you should learn:

How to identify even and odd functions

A graph is symmetric with respect to the y-axis if, whenever (x, y) is on the graph, __________________________ is also on the graph. A graph is symmetric with respect to the x-axis if, whenever (x, y) is on the graph, __________________________ is also on the graph. A graph is symmetric with respect to the origin if, whenever (x, y) is on the graph, __________________________ is also on the graph. A graph that is symmetric with respect to the x-axis is:

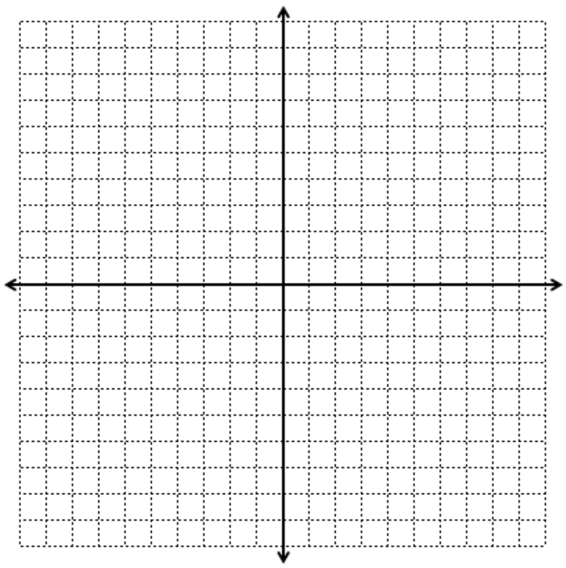
A function is **even** if, for each x in the domain of f, $f(-x) =$ __________________________

A function is **odd** if, for each x in the domain of f, $f(-x) =$ __________________________

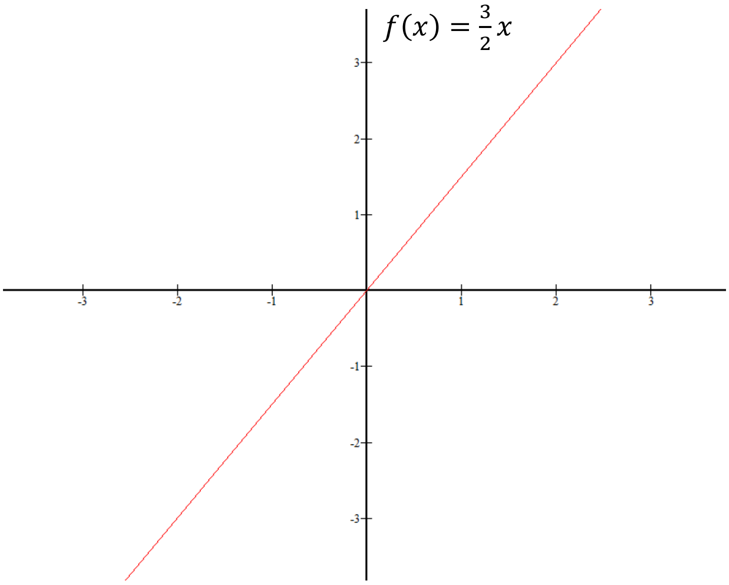
Section 1.3 Examples – Graphs of Functions

(1) Find the domain and range of the function, then graph. Write your domain and range in interval notation.

$$f(x) = \sqrt{4 - x^2}$$



(2)Determine the open interval(s) on which the function is *increasing*, *decreasing*, or *constant*.



(3) Tell whether the function is *even*, *odd*, or *neither* algebraically.

$$f(t) = t^2 + 2t - 3$$

Section 1.4 Shifting, Reflecting, and Stretching Graphs

Objective: In this lesson you learned how to identify and graph shifts, reflections, and nonrigid transformations of functions

Important Vocabulary		
Vertical Shift	**Horizontal Shift**	**Rigid Transformation**
Non-rigid Transformation		

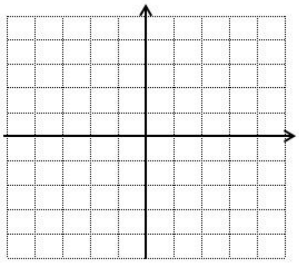
I. Summary of Graphs of Parent Functions

What you should learn:

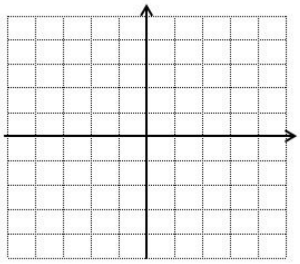
How to recognize graphs of parent functions

Sketch an example of each of the six most commonly used **parent functions** in Algebra.

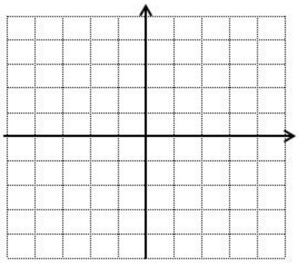
Linear Function



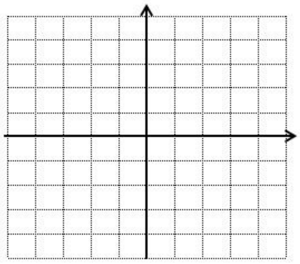
Quadratic Function



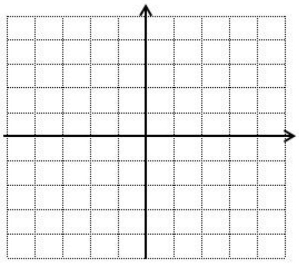
Absolute Value Function



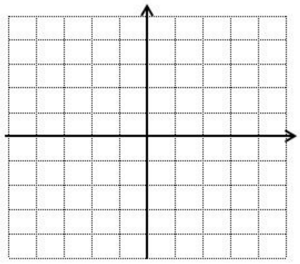
Cubic Function



Square Root Function



Rational Function



II. Vertical and Horizontal Shifts

What you should learn:

How to use vertical and horizontal shifts to graph functions

Let c be a positive real number. Complete the following representations of shifts in the graph of $y = f(x)$:

1) **Vertical shift** c units upward: ________________________________

2) **Vertical shift** c units downward: ______________________________

3) **Horizontal shift** c units to the right: ___________________________

4) **Horizontal shift** c units to the left: ____________________________

III. Reflecting Graphs

What you should learn:

How to use reflections to graph functions

A **reflection** in the x-axis is a type of transformation of the graph of $y = f(x)$ represented by $h(x) =$ ______________.

A **reflection** in the y-axis is a type of transformation of the graph $y = f(x)$ represented by

$h(x) =$ ______________.

IV. Non-rigid Transformations

What you should learn:

How to use nonrigid transformations to graph functions

Name three types of **rigid transformations**:

1)

2)

3)

Rigid transformations change only the ___________________ of the graph in the coordinate plane.

Name four types of **non-rigid transformations**:

1)

2)

3)

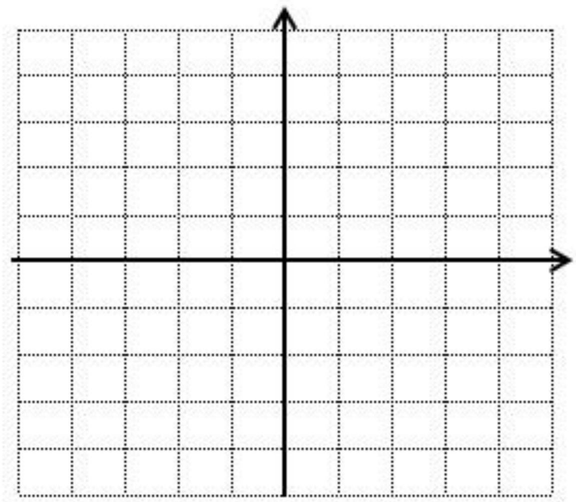
4)

A non-rigid transformation $y = c\,f(x)$ of the graph $y = f(x)$ is a __________________________ if $c > 1$ or a __________________________ if $0 < c < 1$. A nonrigid transformation $y = f(c\,x)$ of the graph of $y = f(x)$ is a __________________________ if $c > 1$ or a __________________________ if $0 < c < 1$.

Section 1.4 Examples – Shifting, Reflecting, and Stretching Graphs

(1) Sketch the graph of the 3 functions on the same rectangular coordinate system.

$f(x) = x$
$g(x) = x - 4$
$h(x) = x + 2$

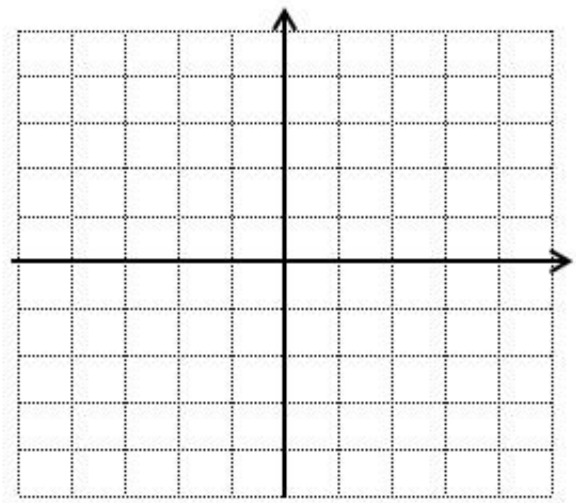


(2) Compare the graph of $g(x)$ with the graph of its parent function.

$g(x) = |x + 1| - 2$

(3) Sketch the graph of the 3 functions on the same rectangular coordinate system.

$f(x) = \sqrt{x}$
$g(x) = \sqrt{-x}$
$h(x) = -\sqrt{x}$



(4) Compare the graph of $g(x)$ with the graph of its parent function.

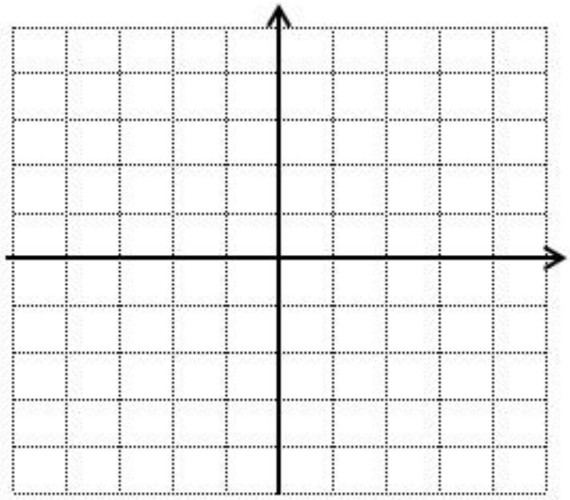
$g(x) = -(x + 2)^3 + 1$

(5) Sketch the graph of the 3 functions on the same rectangular coordinate system.

$f(x) = x^2$

$g(x) = 2(x + 1)^2$

$h(x) = \frac{1}{2}x^2$



Section 1.5 Combinations of Functions

Objective: In this lesson you learned how to find arithmetic combinations and compositions of functions

Important Vocabulary	
Combinations of Functions	**Function Compositions**

I. Arithmetic Combinations of Functions

What you should learn:

How to add, subtract, multiply and divide functions

Just as two real numbers can be combined by the operations of additions, subtraction, multiplication, and division to form other real numbers, two functions f and g can be combined to create new functions such as the ____ of f and g to create new functions.

The domain of an arithmetic combination of functions f and g consists of:

Let f and g be two functions with overlapping domains. Complete the following arithmetic combinations of f and g for all x common to both domains:

1) Sum: $(f + g)(x) =$ ________________________
2) Difference: $(f - g)(x) =$ ________________________
3) Product: $(fg)(x) =$ __________________________
4) Quotient: $\left(\frac{f}{g}\right)(x) =$ ___________________________

II. Compositions of Functions

What you should learn:

How to find compositions of one functions with another function

The **composition** of the function f with the function g is

$(f \circ g)(x) =$ ________________________

For the composition of the function f with g, the domain of $f \circ g$ is:

III. Applications of Combinations of Functions

What you should learn:

How to use combinations of functions to model and solve real-life problems

The function $f(x) = 0.06x$ represents the sales tax owed on a purchase with a price tag of x dollars and the function $g(x) = 0.75x$ represents the sale price of an item with a price tag of x dollars during a 25% off sale. Using one of the combinations of functions discussed in this section, write the function that represents the sales tax owed on an item with a price tag of x dollars during a 25% off sale.

Section 1.5 Examples – Combinations of Functions

(1) Find $\left(\frac{f}{g}\right)(x)$. What is the domain of $\frac{f}{g}$?

$f(x) = 2x - 5 \qquad g(x) = 1 - x$

(2) Determine the domains of (a) f, (b) g, and (c) $f \circ g$

$f(x) = \sqrt{x+3} \qquad g(x) = \frac{x}{2}$

Section 1.6 Inverse Functions

Objective: In this lesson you learned how to find inverse functions graphically and algebraically

Important Vocabulary		
Inverse Function	**One-to-one**	**Horizontal Line Test**

I. Inverse Function

For a function f that is defined by a set of ordered pairs, to form the inverse function of f:

What you should learn:

How to find inverse functions informally and verify that two functions are inverse functions of each other

For a function f and its inverse f^{-1}, the domain of f is equal to ___________________________, and the range of f is equal to ___________________________.

To verify that two functions, f and g, are **inverse functions** of each other:

II. The Graph of an Inverse Function

If the point (a, b) lies on the graph of f, then the point (_____ , _____) lies on the graph of f^{-1} and vice versa. The graph of f^{-1} is a reflection of the graph of f in the line ______________.

What you should learn:

How to use graphs of functions to decide whether functions have inverse functions

III. The Existence of an Inverse Function

If a function is **one-to-one**, that means:

What you should learn:

How to determine if functions are one-to-one

A function f has an inverse f^{-1} if and only if:

To tell whether a function is one-to-one from its graph:

IV. Finding Inverse Functions Algebraically

To find the inverse of a function f algebraically:

1)

2)

3)

4)

5)

What you should learn:

How to find inverse functions algebraically

Section 1.6 Examples – Inverse Functions

(1) Show that f and g are inverse functions (a) algebraically and (b) numerically.

$f(x) = \frac{x-9}{4}$ $\qquad$ $g(x) = 4x + 9$

(2) Determine whether the given function is one-to-one. If so, then find the functions inverse.

$f(x) = 3x + 5$