Chapter 1 – Functions and Their Graphs

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**Vocabulary**

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Section 1.1  Lines in the Plane

Objective: In this lesson you will review how to find and use the slope of a line to write and graph linear equations

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| Linear Function |

I.  The Slope of a Line

The formula for the slope of a line passing through the points \((x_1, y_1)\) and \((x_2, y_2)\) is \(m = \frac{y_2 - y_1}{x_2 - x_1}\).

A line whose slope is positive \__________\ from left to right.

A line whose slope is negative \__________\ from left to right.

A line with zero slope is \__________\.

A line with undefined slope is \__________\.

II. The Point-Slope Form of the Equation of a Line

The point-slope form of the equation of a line is

\[ y - y_1 = m(x - x_1) \]

This form of equation is best used to find the equation of a line when:

The two-point form of the equation of a line is \__________\.

The two-point form of a line is best used to find the equation of a line when:
A linear function has the form _______________________. Its graph is a _______________ that has slope _____ and a y-intercept at (____, ____).

III. Sketching Graphs of Lines

The slope-intercept form of the equation of a line is ______________________, where $m$ is the _______________ and the y-intercept is (____, ____).

The equation of a horizontal line is ______________. The slope of a horizontal line is ______. The y-coordinate of every point on the graph of a horizontal line is ______.

The equation of a vertical line is ______________. The slope of a vertical line is ______________________. The x-coordinate of every point on the graph of a vertical line is ______.

The general form of the equation of a line is _________________.

Every line has an equation that can be written in _________________________.

IV. Parallel and Perpendicular Lines

The relationship between the slope of two lines that are parallel is:

The relationship between the slope of two lines that are perpendicular is:

A line that is parallel to a line whose slope is 2 has a slope of ______.

A line that is perpendicular to a line whose slope is 2 has a slope of ______.
Section 1.1 Examples – Lines in the Plane

( 1 ) Find the point-slope form of the equation that passes through the given point and has the indicated slope.

\((-3, 6)\) \(\quad m = -2\)

( 2 ) Decide whether the two lines are parallel, perpendicular, or neither.

\[ y = 4x - 1 \quad 2x + 8y = 12 \]

( 3 ) Determine the slope and y-intercept (if possible) of the linear equation.

\[ 3x + 4y = 1 \]
Section 1.1.5  Domain and Range

Objective: In this lesson you learned how to identify domain, range, and how to write domain and range in Roster notation, Set-Builder notation, and Interval notation.

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I. Domain and Range

What is domain?

What is range?

How can you find (determine) domain and range when given a graph?

How can you find (determine) domain and range when given an equation?
II. Notation
   A. Roster Notation –

   B. Set-Builder Notation –

   C. Interval Notation –

III. Domain Restrictions
   What causes domain restrictions?
Section 1.1.5 Examples – Domain and Range

(1) Determine the domain and range of the following set of ordered pairs.

\{ (-8, 0), (6, 4), (0, 0), (-7, -3), (10, -5) \}

(2) Given the inequality, write its equivalent form in set-builder notation.

\[ a > 12 \]

(3) Given the inequality, write its equivalent form in interval notation.

\[ x \leq 7 \]

(4) Use the graph of the function to write the domain and range in set-builder notation and interval notation.
Section 1.2 Functions

Objective: In this lesson you learned how to evaluate functions and find their domains

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I. Introduction to Functions

A rule of correspondence that matches quantities from one set with items from a different set is a(n) ________________.

In functions that can be represented by ordered pairs, the first coordinate in each ordered pair is the ________________ and the second coordinate is the ________________.

Some characteristics of a function from Set A to Set B are:

1) ________________
2) ________________
3) ________________
4) ________________

To determine whether or not a relation is a function:

If any input value of a relation is matched with two or more output values:
II. **Function Notation**

The symbol _______ is **function notation** for the value of \( f \) at \( x \) or simply \( f \) of \( x \). The symbol \( f(x) \) corresponds to the ______________________ for a given \( x \).

Keep in mind that ______ is the name of the function, whereas ______ is the output value of the function at the input value \( x \).

In function notation, the __________is the independent variable and the ______________is the dependent variable.

A piecewise-defined function is:

III. **The Domain of a Function**

The **implied domain** of a function defined by an algebraic expression is:

In general, the domain of a function excludes values that:

IV. **Difference Quotients**

A **difference quotient** is defined as:
Section 1.2 Examples – Functions

(1) Evaluate the function at each specified value of the independent variable and simplify.

\[ f(a) = 3a + 5 \]

a) \( f(-2) \)  
b) \( f(4) \)  
c) \( f(x + 1) \)

(2) Find the domain of the function.

\[ s(y) = \frac{3y}{y+5} \]

(3) Find the difference quotient and simplify your answer.

\[ g(x) = 3x - 1 \]

where \( \frac{g(x+h) - g(x)}{h}, h \neq 0 \) is the difference quotient
Section 1.3  Graphs of Functions

Objective: In this lesson you learned how to analyze the graphs of functions

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I. The Graph of a Function

Explain the use of open or closed dots in the graphs of functions:

To find the domain of a function from its graph:

To find the range of a function from its graph:

The Vertical Line Test for functions states:

II. Increasing and Decreasing Functions

A function $f$ is **increasing** on an interval if, for any $x_1$ and $x_2$ in the interval:

What you should learn:

How to find the domains and ranges of functions and how to use the Vertical Line Test for functions

How to determine intervals on which functions are increasing, decreasing, or constant
A function $f$ is **decreasing** on an interval if, for any $x_1$ and $x_2$ in the interval:

A function $f$ is **constant** on an interval if, for any $x_1$ and $x_2$ in the interval:

Given a graph of a function, how do you determine when a function is *increasing, decreasing, or constant*?

### III. Relative Minimum and Maximum Values

A function value $f(a)$ is called a **relative minimum** of $f$ if:

A function value $f(a)$ is called a **relative maximum** of $f$ if:

The point at which a function changes from increasing to decreasing is a relative _________________. The point at which a function changes from decreasing to increasing is a relative _________________.

What you should learn:

- How to determine relative minimum and relative maximum values of functions
IV. Step Functions and Piecewise-Defined Functions

Describe the graph of the greatest integer function:

Describe the graph of a piecewise-defined function:

V. Even and Odd Functions

A graph is symmetric with respect to the y-axis if, whenever $(x, y)$ is on the graph, _________________________ is also on the graph. A graph is symmetric with respect to the x-axis if, whenever $(x, y)$ is on the graph, _________________________ is also on the graph. A graph is symmetric with respect to the origin if, whenever $(x, y)$ is on the graph, _________________________ is also on the graph. A graph that is symmetric with respect to the x-axis is:

A function is **even** if, for each $x$ in the domain of $f$, $f(-x) = _________________________$

A function is **odd** if, for each $x$ in the domain of $f$, $f(-x) = _________________________$
Section 1.3 Examples – Graphs of Functions

(1) Find the domain and range of the function, then graph. Write your domain and range in interval notation.

\[ f(x) = \sqrt{4 - x^2} \]

(2) Determine the open interval(s) on which the function is increasing, decreasing, or constant.

\[ f(x) = \frac{3}{2}x \]

(3) Tell whether the function is even, odd, or neither algebraically.

\[ f(t) = t^2 + 2t - 3 \]
Section 1.4 Shifting, Reflecting, and Stretching Graphs

Objective: In this lesson you learned how to identify and graph shifts, reflections, and nonrigid transformations of functions.

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<td>Horizontal Shift</td>
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I. Summary of Graphs of Parent Functions

Sketch an example of each of the six most commonly used parent functions in Algebra.

- Linear Function
- Quadratic Function
- Absolute Value Function
- Cubic Function
- Square Root Function
- Rational Function

What you should learn:

How to recognize graphs of parent functions.
II. **Vertical and Horizontal Shifts**

Let $c$ be a positive real number. Complete the following representations of shifts in the graph of $y = f(x)$:

1) **Vertical shift** $c$ units upward: _________________________________

2) **Vertical shift** $c$ units downward: _______________________________

3) **Horizontal shift** $c$ units to the right: __________________________

4) **Horizontal shift** $c$ units to the left: ___________________________

III. **Reflecting Graphs**

A **reflection** in the x-axis is a type of transformation of the graph of $y = f(x)$ represented by $h(x) = $ ____________.

A **reflection** in the y-axis is a type of transformation of the graph $y = f(x)$ represented by $h(x) = $ ____________.

IV. **Non-rigid Transformations**

Name three types of **rigid transformations**:  

1) 

2) 

3) 

Rigid transformations change only the ____________________ of the graph in the coordinate plane.
Name four types of non-rigid transformations:

1) 

2) 

3) 

4) 

A non-rigid transformation $y = cf(x)$ of the graph $y = f(x)$ is a ____________________ if $c > 1$ or a ____________________ if $0 < c < 1$. A nonrigid transformation $y = f(cx)$ of the graph of $y = f(x)$ is a ____________________ if $c > 1$ or a ____________________ if $0 < c < 1$. 
Section 1.4 Examples – Shifting, Reflecting, and Stretching Graphs

(1) Sketch the graph of the 3 functions on the same rectangular coordinate system.

\[ f(x) = x \]
\[ g(x) = x - 4 \]
\[ h(x) = x + 2 \]

(2) Compare the graph of \( g(x) \) with the graph of its parent function.

\[ g(x) = |x + 1| - 2 \]

(3) Sketch the graph of the 3 functions on the same rectangular coordinate system.

\[ f(x) = \sqrt{x} \]
\[ g(x) = \sqrt{-x} \]
\[ h(x) = -\sqrt{x} \]

(4) Compare the graph of \( g(x) \) with the graph of its parent function.

\[ g(x) = -(x + 2)^3 + 1 \]
(5) Sketch the graph of the 3 functions on the same rectangular coordinate system.

\[ f(x) = x^2 \]
\[ g(x) = 2(x + 1)^2 \]
\[ h(x) = \frac{1}{2}x^2 \]
Section 1.5 Combinations of Functions

Objective: In this lesson you learned how to find arithmetic combinations and compositions of functions

I. Arithmetic Combinations of Functions

Just as two real numbers can be combined by the operations of additions, subtraction, multiplication, and division to form other real numbers, two functions \( f \) and \( g \) can be combined to create new functions such as the ___________________________ of \( f \) and \( g \) to create new functions.

The domain of an arithmetic combination of functions \( f \) and \( g \) consists of:

Let \( f \) and \( g \) be two functions with overlapping domains. Complete the following arithmetic combinations of \( f \) and \( g \) for all \( x \) common to both domains:

1) Sum: \((f + g)(x) = \) _______________________
2) Difference: \((f - g)(x) = \) _______________________
3) Product: \((fg)(x) = \) _______________________
4) Quotient: \((\frac{f}{g})(x) = \) _______________________

II. Compositions of Functions

The composition of the function \( f \) with the function \( g \) is 
\((f \circ g)(x) = \) _______________________

What you should learn:
How to add, subtract, multiply and divide functions
How to find compositions of one functions with another function
For the composition of the function $f$ with $g$, the domain of $f \circ g$ is:

III. Applications of Combinations of Functions

The function $f(x) = 0.06x$ represents the sales tax owed on a purchase with a price tag of $x$ dollars and the function $g(x) = 0.75x$ represents the sale price of an item with a price tag of $x$ dollars during a 25% off sale.

Using one of the combinations of functions discussed in this section, write the function that represents the sales tax owed on an item with a price tag of $x$ dollars during a 25% off sale.
Section 1.5 Examples – Combinations of Functions

(1) Find \( f \circ g \)(\( x \)). What is the domain of \( f \circ g \)?
\[
f(x) = 2x - 5 \quad g(x) = 1 - x
\]

(2) Determine the domains of (a) \( f \), (b) \( g \), and (c) \( f \circ g \)
\[
f(x) = \sqrt{x + 3} \quad g(x) = \frac{x}{2}
\]
Section 1.6 Inverse Functions

Objective: In this lesson you learned how to find inverse functions graphically and algebraically.

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I. Inverse Function

For a function $f$ that is defined by a set of ordered pairs, to form the inverse function of $f$: 

For a function $f$ and its inverse $f^{-1}$, the domain of $f$ is equal to $\ldots$ and the range of $f$ is equal to $\ldots$.

To verify that two functions, $f$ and $g$, are inverse functions of each other:

II. The Graph of an Inverse Function

If the point $(a, b)$ lies on the graph of $f$, then the point $(\ldots, \ldots)$ lies on the graph of $f^{-1}$ and vice versa. The graph of $f^{-1}$ is a reflection of the graph of $f$ in the line $\ldots$.

III. The Existence of an Inverse Function

If a function is one-to-one, that means:

A function $f$ has an inverse $f^{-1}$ if and only if:
To tell whether a function is one-to-one from its graph:

IV. Finding Inverse Functions Algebraically

To find the inverse of a function $f$ algebraically:

1) 

2) 

3) 

4) 

5)
Section 1.6 Examples – Inverse Functions

(1) Show that \( f \) and \( g \) are inverse functions (a) algebraically and (b) numerically.

\[
f(x) = \frac{x-9}{4} \quad \quad \quad g(x) = 4x + 9
\]

(2) Determine whether the given function is one-to-one. If so, then find the functions inverse.

\[
f(x) = 3x + 5
\]