Section 8.4 The Binomial Theorem

Objective: In this lesson you learned how to use the Binomial Theorem and Pascal's Triangle to calculate binomial coefficients and write binomial expansions.

Important Vocabulary		
Binomial coefficients	**The Binomial Theorem**	**Pascal's Triangle**

I. Binomial Coefficients

What you should learn:

How to use the Binomial Theorem to calculate binomial coefficients

List four general observations about the expansion of $(x+y)^n$ for various values of n.

1. In each expansion, there are $n+1$ terms
   $(x+2)^2 = x^2+4x+4$
   Binomial -> trinomial
2. In each expansion, x and y have symmetric roles. In successive terms, the power of x decreases by 1 and the power of y increases by 1.
3. The sum of powers in each term is n
4. The coefficients increase then decrease in a symmetric pattern.

The **Binomial Theorem** states that in the expansion of $(x+y)^n = x^n + nx^{n-1}y + \cdots +$

${}_nC_r x^{n-r}y^r + \cdots + nxy^{n-1} + y^n$, the coefficient of $x^{n-r}y^r$ is:

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

II. Binomial Expansion

What you should learn:

How to use binomial coefficients to write binomial expansions

Writing out the coefficients for a binomial that is raised to a power is called expanding a binomial

III. Pascal's Triangle

What you should learn:

How to use Pascal's Triangle to calculate binomial coefficients

Construct rows 0 through 6 of Pascal's Triangle.

1 ← row 0
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1 ← row 6

Section 8.4 Examples – The Binomial Theorem

(1) Find the binomial coefficient ${}_{12}C_5$.

$${}_{12}C_5 = \frac{12!}{(12-5)!\,5!} = 792$$

(2) Write the expansion of the expression $(x+2)^5$

exponent of 5 ← 5th row of Pascal's Triangle:

$$(x+2)^5 = 1(x^5) + 5(2)(x^4) + 10(2^2)(x^3) + 10(2^3)(x^2) + 5(2^4)(x) + 1(2^5)$$

$$= x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

can also calculate as ${}_5C_3 = 10$