

Section 6.2 Law of Cosines

Objective: In this lesson you learned how to use the Law of Cosines to solve oblique triangles and to use Heron's Formula to find the area of a triangle.

Important Vocabulary

Law of Cosines

I. Introduction

State the Law of Cosines

If ABC is a triangle with sides a, b and c ,
then

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

When given the lengths of all three sides of a triangle and asked to find all three angles, which angle should be found first? Why?

The largest angle should be found first so you can determine if it is acute or obtuse.

What you should learn:

How to use the Law of Cosines to solve oblique triangles (SSS or SAS)

II. Heron's Area Formula

Heron's Area Formula states that given any triangle with sides of length a, b , and c , the area of the triangle is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ where}$$

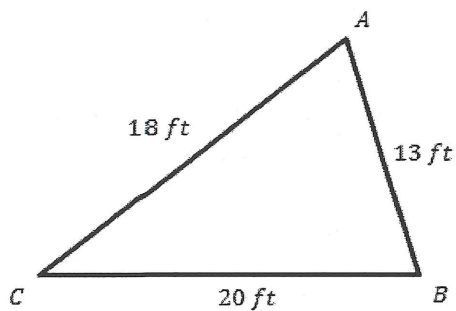
$$s = \frac{a+b+c}{2}$$

What you should learn:

How to use Heron's Area Formula to find areas of triangles

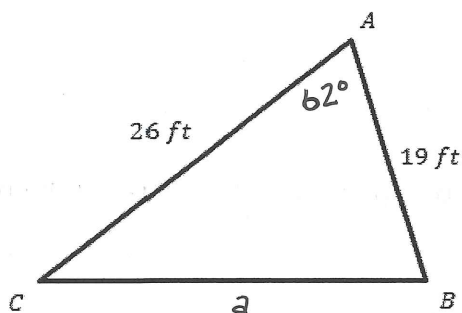
Section 6.2 Examples – Law of Cosines

(1) Using the triangle shown below, find angle A (in degrees).



$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A \\20^2 &= 18^2 + 13^2 - 2(18)(13) \cos A \\-93 &= -468 \cos A \\0.1987 &= \cos A \\A &= 78.5^\circ\end{aligned}$$

(2) In the triangle shown below, if $A = 62^\circ$, find the length of side a .



$$\begin{aligned}a^2 &= 26^2 + 19^2 - 2(26)(19) \cos 62^\circ \\a^2 &= 573.16 \text{ ft}^2 \\a &= 23.9 \text{ ft}\end{aligned}$$

(3) Find the area of a triangle having sides of length $a = 14 \text{ cm}$, $b = 21 \text{ cm}$, and $c = 27 \text{ cm}$.

$$\begin{aligned}s &= \frac{14 + 21 + 27}{2} \\A &= \sqrt{31(31-14)(31-21)(31-27)} \\&= 145.2 \text{ cm}^2\end{aligned}$$