Section 6.1 Law of Sines

Objective: In this lesson you learned how to use the Law of Sines to solve oblique triangles and how to find the areas of oblique triangles.

Important Vocabulary

Oblique triangle — A triangle without a right angle.	**Acute Triangle**	**Obtuse Triangle**	**Law of Sines**

I. Introduction

What you should learn:

How to use the Law of Sines to solve oblique triangles (AAS or ASA)

State the **Law of Sines**

If ABC is a triangle with sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

To solve an oblique triangle, you need to know the measure of at least one side and any two other parts of the triangle. Describe two cases (think information given about a triangle) that can be solved using the Law of Sines.

Two angles and any side (AAS or ASA)

Two sides and an angle opposite one of the sides (SSA)

leads to this.

II. The Ambiguous Case (SSA)

What you should learn:

How to use the Law of Sines to solve oblique triangles (SSA)

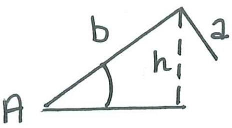
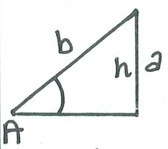
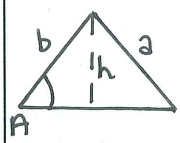
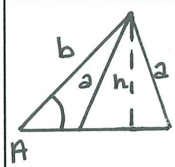
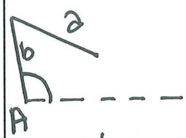
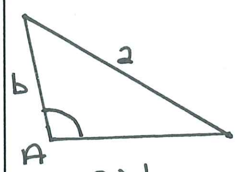
If two sides and one opposite angle of an oblique triangle are given, three possible situations can occur, which are:

1) No triangle exists

2) one triangle exists

3) two possible triangles.

Complete the table from p. 406 of your textbook.

A is acute	A is acute	A is acute	A is acute	A is obtuse	A is obtuse
					
$a < h$	$a = h$	$a \geq b$	$h < a < b$	$a \leq b$	$a > b$
NO triangle.	one triangle	one triangle	two triangles	NO Triangle.	one triangle.

side adjacent to the angle.

NOTE! $h = b \sin A$ when given a, b and A

angle.

III. Area of an Oblique Triangle

The area of any triangle is one-half (½) the product of the lengths of two sides times the sine of their included angle. That is,

$Area = \frac{1}{2} bc \sin A$, or

$Area = \frac{1}{2} ac \sin B$, or

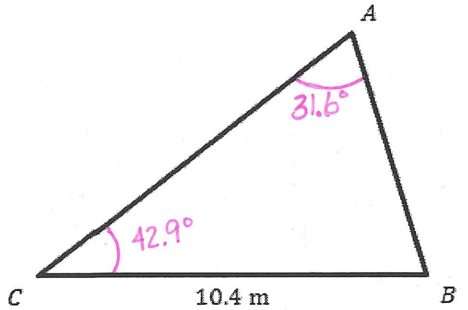
$Area = \frac{1}{2} ab \sin C$

What you should learn:

How to find areas of oblique triangles and use the Law of Sines to model and solve real-life problems

Section 6.1 Examples – Law of Sines

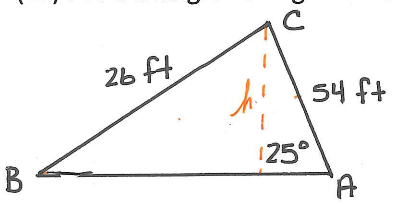
(1) For the triangle shown below, $A = 31.6°$, $C = 42.9°$, and $a = 10.4$ meters. Find the length of side c.



$$\frac{c}{\sin 42.9°} = \frac{10.4\text{ m}}{\sin 31.6°}$$

$\boxed{c = 13.5\text{ m}}$

(2) For a triangle having $A = 25°, b = 54$ feet, and $a = 26$ feet, how many solutions are possible?



* Two sides, and an opposite angle *

$h = 54 \sin 25°$

$h = 22.8$ ft

$h < a < b$ ✓

$\boxed{\text{2 triangles possible}}$

(3) Use the Law of Sines to solve the triangle. If two solutions exist, find both.

$A = 58°, a = 11.4, b = 12.8$

$h = 12.8 \sin 58°$

$h = 10.9$

$h < a < b$

2 △s ✓

$$\frac{\sin B_1}{12.8} = \frac{\sin 58°}{11.4}$$

$\boxed{B_1 = 72°}$

$\boxed{C = 50°}$

$$\frac{c}{\sin 50°} = \frac{11.4}{\sin 58°}$$

$\boxed{c = 10.3}$

$B_2 = 180° - 72°$

$\boxed{B_2 = 108°}$

$\boxed{C = 14°}$

$$\frac{c}{\sin 14°} = \frac{11.4}{\sin 58°}$$

$\boxed{c = 3.3}$

(4) Find the area of a triangle having two sides of lengths 30 feet and 48 feet and an included angle of 40°.

$A = \frac{1}{2}(30)(48)\sin 40°$

$= 462.8\text{ ft}^2$