Section 5.4 Sum and Difference Formulas

Objective: In this lesson you learned how to use sum and difference formulas to rewrite and evaluate trigonometric functions.

Important Vocabulary	
Sum and Difference Formulas	Reduction Formulas

How to use sum and difference formulas to evaluate trigonometric functions, to verify identities and to solve trigonometric equations

I. Using Sum and Difference Formulas

List the **sum and difference formulas** for sine, cosine, and tangent.

$\sin(u+v) =$ $\sin u \cos v + \cos u \sin v$

$\sin(u-v) =$ $\sin u \cos v - \cos u \sin v$

$\sin(u \pm v)$

$\cos(u+v) =$ $\cos u \cos v - \sin u \sin v$

$\cos(u-v) =$ $\cos u \cos v + \sin u \sin v$

$\cos(u \pm v)$

$\tan(u+v) =$ $\dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

$\tan(u-v) =$ $\dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

$\tan(u \pm v)$

A **reduction formula** is:

a formula involving expressions such as $\sin\left(\theta + \frac{n\pi}{2}\right)$ or $\cos\left(\theta + \frac{n\pi}{2}\right)$, where n is an integer, that can be derived from sum and difference formulas

Section 5.4 Examples – Sum and Difference Formulas

(1) Find the exact value of each expression.

a) $\cos(240° - 0°)$

$= \cos 240° \cos 0° + \sin 240° \sin 0°$

$= -\frac{1}{2}(1) + -\frac{\sqrt{3}}{2}(0)$

$\boxed{= -\frac{1}{2}}$

b) $\cos 240° - \cos 0°$

$= -\frac{1}{2} - 1$

$\boxed{= -\frac{3}{2}}$

You CANNOT "DISTRIBUTE" Trigonometric functions!

(2) Find the exact values of the sine, cosine, and tangent of the angle.

$$165° = 135° + 30°$$

$\sin 165° = \sin(135° + 30°) = \sin 135° \cos 30° + \cos 135° \sin 30°$

$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) + -\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$\cos 165° = \cos(135° + 30°) = \cos 135° \cos 30° - \sin 135° \sin 30°$

$= -\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

$\tan 165° = \tan(135° + 30°) = \dfrac{\tan 135° + \tan 30°}{1 - \tan 135° \tan 30°}$

$= \dfrac{-1 + \frac{\sqrt{3}}{3}}{1 - (-1)\left(\frac{\sqrt{3}}{3}\right)} = \boxed{\dfrac{-3+\sqrt{3}}{3+\sqrt{3}}}$

(3) Write the expression as the sine, cosine, or tangent of an angle.

$$\cos 60° \cos 10° - \sin 60° \sin 10°$$

$\cos(60° + 10°)$

$\boxed{\cos 70°}$

(4) Find the exact value of the expression without using a calculator.

$$\sin\left[\frac{\pi}{2} + \boxed{\sin^{-1}(-1)}\right]$$

oops... INVERSE SINE... So, WHAT ANGLE gives you A sine value of -1?

$\sin\left[\frac{\pi}{2} + \frac{3\pi}{2}\right]$

$\sin 2\pi$

$\boxed{0}$