Section 5.3 Trees and Counting Techniques

Objective: In this lesson you learned to use tree diagrams, compute number of ordered arrangements, and nonordered groupings.

Important Vocabulary

Multiplication Rule of Counting	Tree Diagram	Factorial Notation
Permutation	Combination	

Focus Points:

- Organize outcomes in a sample space using tree diagrams
- Compute the number of ordered arrangements of outcomes using permutations
- Compute number of (nonordered) groupings of outcomes using combinations
- Explain how counting techniques relate to probability in everyday life

What formula is used, when outcomes are equally likely, to compute the probability of an event?

$$P(A) = \frac{\#\text{ favorable outcomes}}{\#\text{ of outcomes in sample space}}$$

Multiplication Rule of Counting

$n_1 \times n_2 \times \cdots n_m =$ total # of outcomes for a series of Events $E_1, E_2, \ldots, E_m$ where $n_1, n_2, \ldots, n_m$ are the number of possibilities for each event.

What is a **tree diagram**?

Gives a visual display of the total number of outcomes of an experiment consisting of a series of events.

Factorial Notation

$0! = 1$

$1! = 1$

$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$

Permutations

The number of ways to arrange in order n distinct objects, r at a time

$$P_{n,r} = {}_nP_R = \frac{n!}{(n-r)!}$$

ORDER MATTERS!

Combinations

The number of combinations of n objects, taken r at a time

$$C_{n,r} = \binom{n}{r} = {}_nC_r = \frac{n!}{r!\,(n-r)!}$$

ORDER DOES NOT MATTER!

What are the differences between combinations and permutations?

Permutations → groupings AND order

Combinations → only number of groupings

How to determine the number of outcomes of an experiment.

1. If the experiment consists of a series of stages with various outcomes, use the multiplication rule/tree diagram.

2. If the outcomes consist of ordered subgroups

$${}_nP_r = \frac{n!}{(n-r)!}$$

3. If the outcomes consist of nonordered subgroups

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$

What do counting rules tell us?

The total number of outcomes created by combining a sequence of events in specified ways.

- The multiplication rule tells us the total number of possible outcomes. Tree diagrams provide a visual display.
- Permutation Rule → total number of ways to arrange in order
- Combination Rule → How many ways to be formed into groups.

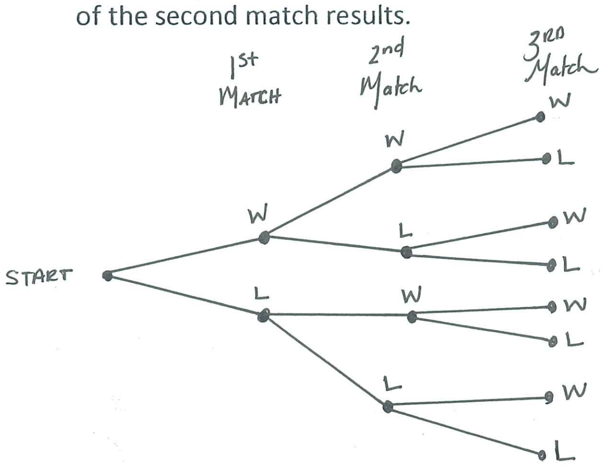
Section 5.3 – Trees and Counting Techniques

(1) Louis played three tennis matches. Use a tree diagram to list the possible win and loss sequences Louis can experience for the set of three matches.

a. On the first match Louis can win or lose. From Start, indicate these two branches.

b. Regardless of whether Louis wins or loses the first match, he plays the second and can again win or lose. Attach branches representing these two outcomes to *each* of the first match results.

c. Louis may win or lose the third match. Attach braches representing these two outcomes to *each* of the second match results.



d. How many possible win-lose sequences are there for the three matches?

8 sequences

e. Complete the list of win-lose sequences.

1st	2nd	3rd
W	W	W
W	W	L
W	L	W
W	L	L
L	W	W
L	W	L
L	L	W
L	L	L

(2) The board of directors at Belford Community Hospital has 12 members. Three officers – president, vice president, and treasurer – must be elected from among the members. How many different slates of officers are possible? We will view a slate of officers as a list of three people, with the president listed first, the vice president listed second, and the treasurer listed third. Not only are we asking for the number of different groups of three names for a slate, we are also concerned about order.

a. Do we use the permutations rule or the combinations rule? What is the value of n? What is the value of r?

Permutations – President, Vice President, Treasurer
↳ Specific Order

b. Use the permutations rule with $n = 12$ and $r = 3$ to compute ${}_{12}P_3$.

$${}_{12}P_3 = \frac{12!}{(12-3)!} = 1320$$

Three members from the group of 12 on the board of directors at Belford Community Hospital will be selected to go to a convention (all expenses paid) in Hawaii. How many different groups of 3 are there?

c. Do we use the permutations rule or the combinations rule? What is the value of n? What is the value of r?

Combinations – No order.

d. Use the combinations rule with $n = 12$ and $r = 3$ to compute ${}_{12}C_3$.

$${}_{12}C_3 = \frac{12!}{3!\,(12-3)!} = 220$$