Section 5.2 Some Probability Rules – Compound Events

Objective: In this lesson you will learn to compute probabilities of general compound events, independent events, and compute conditional probabilities.

Important Vocabulary		
Independent Event	Dependent Event	Conditional Probability
Mutually Exclusive	Disjoint	

Focus Points:

- Compute probabilities of general compound events

I. Conditional Probability and Multiplication Rules

Two events are **independent:** if the occurrence or nonoccurrence of one event does not change the probability that the other event will occur.

In **dependent events**, the outcome of the first event:

changes the probability of the second event.

Multiplication Rule for Independent Events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

What is **conditional probability**?

The probability that Event A will occur given that Event B has occurred.

General Multiplication Rule for Any Event

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ "given" $P(A \text{ and } B) = P(B) \cdot P(A|B)$

Conditional Probability (When $P(B) \neq 0$)

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \qquad P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

When do the multiplication rules apply?

whenever you wish to determine the probability of two events happening together

What word indicates *together*?

"And"

How to use the Multiplication Rules

1. Determine if events A and B are independent.
   if $P(A) = P(A|B)$, they are independent

2. If independent
   $P(A \text{ and } B) = P(A) \cdot P(B)$

3. For any events
   $P(A \text{ and } B) = P(A) \cdot P(B|A)$
   $= P(B) \cdot P(A|B)$

What does conditional probability tell us?

- Probability that Event A will happen.

- Probability that Event B will happen, assuming Event A has happened.

- If $P(A|B) = P(A)$ or $P(B|A) = P(B)$, then A and B are independent events

- Conditional Probability
  $P(A \text{ and } B) = P(A) \cdot P(B|A)$
  $= P(B) \cdot P(A|B)$

  if Events are Independent
  $P(A \text{ and } B) = P(A) \cdot P(B)$

- $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$, $P(B) \neq 0$

II. Addition Rules

What is another way to combine events?

Considering one event OR another occurring.

Focus Points:
- Compute probabilities involving independent events or mutually exclusive events
- Use survey results to compute conditional probabilities

What are the ways to satisfy the condition *A or B*?

1. Any outcome in A occurs

2. Any outcome in B occurs

3. Any outcome in Both A and B occurs.

Two events are **mutually exclusive** or: disjoint if they cannot occur together $P(A \text{ and } B) = 0$

Addition Rule for Mutually Exclusive Events *A and B*

$P(A \text{ or } B) = P(A) + P(B)$

General Addition Rule for any events *A and B*

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

How to use the Addition Rules

1. Determine if A and B are mutually exclusive

   if $P(A \text{ and } B) = 0$, then A and B are disjoint

2. Mutually Exclusive Events

   $P(A \text{ or } B) = P(A) + P(B)$

3. Any Events

   $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Summary of Basic Probability Rules

A statistical experiment is any random activity that results in a recordable outcome.

1. P(entire sample space) = 1

2. for any event A: $0 \le P(A) \le 1$

3. Complement of A: $P(A^c) = 1 - P(A)$

4. Independent Events if: $P(A) = P(A|B)$

5. Multiplication Rules

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$
$$= P(B) \cdot P(A|B)$$

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Independent Events

6. $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}, \; P(B) \neq 0$

7. Mutually Exclusive if $P(A \text{ and } B) = 0$.

8. Addition Rules

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B)$$

mutually exclusive

Section 5.2 – Some Probability Rules – Compound Events

(1) Andrew is 55, and the probability that he will be alive in 10 years is 0.72. Ellen is 35, and the probability that she will be alive in 10 years is 0.92. Assuming that the life span of one will have no effect on the life span of the other, what is the probability that they will both be alive in 10 years?

a. Are these events dependent or independent?

Independent

One Life span does not affect the other.

b. Use the appropriate multiplication rule to find $P(\text{Andrew alive in 10 years } and \text{ Ellen alive in 10 years})$.

$P(\text{Andrew } \underline{\text{and}} \text{ Ellen alive in 10 years}) = P(A) \cdot P(E)$

$= (0.72)(0.92)$

$\boxed{= 0.66}$

(2) A quality-control procedure for testing Ready-Flash digital cameras consists of drawing two cameras at random from each lot of 100 without replacing the first camera before drawing the second. If both are defective, the entire lot is rejected. Find the probability that both cameras are defective if the lot contains 10 defective cameras. Since we are drawing the cameras at random, assume that each camera in the lot has an equal chance of being drawn.

a. What is the probability of getting a defective camera on the first draw?

$P(\text{defective camera}) = \frac{10}{100} = \frac{1}{10} = 0.1$

b. The first camera is not replaced, so there are only 99 cameras for the second draw. What is the probability of getting a defective camera on the second draw if the first camera was defective?

$P(\text{defective 2}^{\text{nd}} \text{ camera} \mid \text{defective 1}^{\text{st}} \text{ camera}) = \frac{9}{99} = \frac{1}{11} = 0.\overline{09}$

c. Are the probabilities computed in parts (a) and (b) different? Does drawing a defective camera on the first draw change the probability of getting a defective camera on the second draw? Are the events dependent?

Yes to all.

d. Use the formula for dependent events, $P(A \text{ } and \text{ } B) = P(A) \cdot P(B|A)$ to compute $P(\text{1st camera defective } and \text{ 2nd camera defective})$.

$P(1^{\text{st}} \text{defective } \underline{\text{and}} \text{ 2}^{\text{nd}} \text{defective}) = \frac{1}{10} \cdot \frac{1}{11} = \frac{1}{110} = 0.0\overline{09}$

(3) Indicate how each of the following pairs of events are combined. Use either the *and* combination or the *or* combination.

a. Satisfying the humanities requirement by taking a course in the history of Japan or by taking a course in classical literature.

OR

b. Buying new tires and aligning the tires.

AND

c. Getting an A not only in psychology but also in biology.

AND

d. Having at least one of the pets: cat, dog, bird, rabbit.

OR

(4) The Cost Less Clothing Store carries remainder pairs of slacks. If you buy a pair of slacks in your regular waist size without trying them on, the probability that the waist will be too tight is 0.30 and the probability that it will be too lose is 0.10.

a. Are the events "too tight" and "too loose" mutually exclusive?

Yes. The waist cannot be "too tight" AND "too loose" at the same time.

b. If you choose a pair of slacks at random in your regular waist size, what is the probability that the waist will be too tight or too lose?

$$P(\text{too tight or too Loose}) = P(\text{tight}) + P(\text{loose})$$
$$= 0.3 + 0.1 = \boxed{0.40}$$

(5) Professor Jackson is in charge of a program to prepare people for a high school equivalency exam. Records show that 80% of the students need work in math, 70% need work in English, and 55% need work in both areas.

a. Are the events "needs math" and "needs English" mutually exclusive?

No. Some students need both.

b. Use the appropriate formula to compute the probability that a student selected at random needs math *or* needs English.

$$P(\text{needs Math or needs English}) = P(M) + P(E) - P(B)$$
$$= 0.8 + 0.7 - 0.55$$
$$\boxed{= 0.95}$$

(6) Using the table to the right, let's consider other probabilities regarding the types of employees at Hopewell and their political affiliations. This time let's consider the production worker and the affiliation of Independent. Suppose an employee is selected at random from the group of 140.

Employee Type and Political Affiliation

	Political Affiliation				
Employee Type	Deomcrat (D)	Republican (R)	Independent (I)	Row Total	
Executive (E)	5	34	9	48	
Production Worker (PW)	63	21	8	92	
Column Total	68	55	17	140	Grand Total

a. Compute $P(I)$ and $P(PW)$.

$P(I) = \frac{17}{140} \approx 0.121$ $\qquad$ $P(PW) = \frac{92}{140} \approx 0.657$

b. Compute $P(I|PW)$. This is a conditional probability. Be sure to restrict your attention to production workers, since that is the condition given.

$P(I|PW) = \frac{8}{92} \approx 0.087$

c. Compute $P(I \text{ and } PW)$. In this case, look at the entire sample space and the number of employees who are both Independent and in production.

$P(I \text{ and } PW) = \frac{8}{140} \approx 0.057$

d. Use the multiplication rule for dependent events to calculate $P(I \text{ and } PW)$. Is the result the same as that of part (c)?

$P(I \text{ and } PW) = P(PW) \cdot P(I|PW) = \frac{92}{140} \cdot \frac{8}{92} = \frac{8}{140} \approx 0.057$

Yes they are the same.

e. Compute $P(I \text{ or } PW)$. Are the events mutually exclusive?

$$P(I \text{ or } PW) = P(I) + P(PW) - P(I \text{ and } PW)$$
$$= \frac{17}{140} + \frac{92}{140} - \frac{8}{140}$$
$$= \frac{101}{140} \approx 0.721$$