

Section 4.4 Trigonometric Functions of Any Angle

Objective: In this lesson you learned how to evaluate trigonometric functions of any angle.

Important Vocabulary

Reference Angle

I. Introduction

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$. Complete the following definitions of the trigonometric functions of any angle.

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}, x \neq 0$$

$$\csc \theta = \frac{r}{y}, y \neq 0$$

$$\sec \theta = \frac{r}{x}, x \neq 0$$

$$\cot \theta = \frac{x}{y}, y \neq 0$$

What you should learn:

How to evaluate trigonometric functions of any angle

Name the quadrant(s) in which the sine function is positive: I II

Name the quadrant(s) in which the sine function is negative: III IV

Name the quadrant(s) in which the cosine function is positive: I IV

Name the quadrant(s) in which the cosine function is negative: II III

Name the quadrant(s) in which the tangent function is positive: I III

Name the quadrant(s) in which the tangent function is negative: II IV

II. Reference Angles

The definition of a **Reference Angle** states that:

θ is an angle in standard position. Its reference angle is the acute angle θ' formed by the terminal side and the x-axis

Pronounced "Theta Prime"

What you should learn:

How to use reference angles to evaluate trigonometric functions

How to find a reference angle in each of the following quadrants:

$$\text{II: } \theta' = 180^\circ - \theta \\ = \pi - \theta$$

$$\text{III: } \theta' = \theta - 180^\circ \\ = \theta - \pi$$

$$\text{IV: } \theta' = 360^\circ - \theta \\ = 2\pi - \theta$$

will also use α \leftarrow greek letter Alpha

III. Trigonometric Functions of Real Numbers

To find the value of a trigonometric function of any angle θ , you:

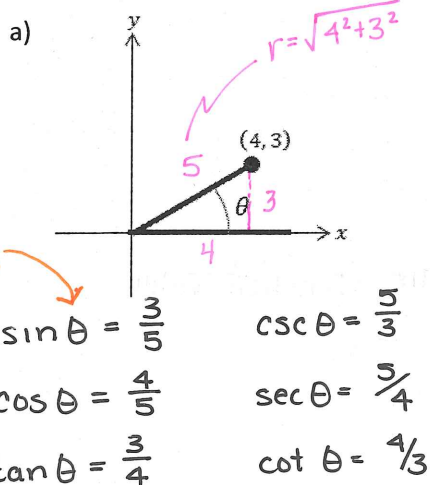
- 1) find the reference angle θ'
- 2) Determine the quadrant θ lies in
- 3) Affix the appropriate sign to the function value
↑ + or -

What you should learn:

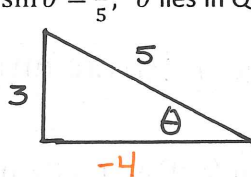
How to evaluate trigonometric functions of real numbers

Section 4.4 Examples – Trigonometric Functions of Any Angle

(1) Determine the exact values of the six trigonometric functions of the angle θ .



b) $\sin \theta = \frac{3}{5}$, θ lies in Quadrant II



$$r = \sqrt{x^2 + y^2}$$

$$5^2 = (-x)^2 + (3)^2$$

$$x = -4$$

$$\sin \theta = \frac{3}{5}$$

$$\csc \theta = \frac{5}{3}$$

$$\cos \theta = \frac{-4}{5}$$

$$\sec \theta = \frac{5}{-4}$$

$$\tan \theta = \frac{3}{-4}$$

$$\cot \theta = \frac{-4}{3}$$

(2) Find the reference angle θ' for the special angle θ .

120° in Quad II

$$\theta = 120^\circ$$

$$\theta' = 180^\circ - 120^\circ = \boxed{60^\circ}$$

(3) Find the exact value for each function for the given angle for $f(\theta) = \sin \theta$ and $g(\theta) = \cos \theta$.

$$\theta = 30^\circ$$

a) $(f + g)(\theta)$

$$= \sin 30^\circ + \cos 30^\circ = \frac{1}{2} + \frac{\sqrt{3}}{2} = \boxed{\frac{1 + \sqrt{3}}{2}}$$

b) $(g - f)(\theta)$

$$= \cos 30^\circ - \sin 30^\circ = \frac{\sqrt{3}}{2} - \frac{1}{2} = \boxed{\frac{\sqrt{3} - 1}{2}}$$

c) $[g(\theta)]^2$

$$= [\cos 30^\circ]^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \boxed{\frac{3}{4}}$$

d) $(fg)(\theta)$

$$= [\sin 30^\circ][\cos 30^\circ] = \frac{1}{2} \left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{3}}{4}}$$

e) $f(2\theta)$

$$= \sin 60^\circ = \boxed{\frac{\sqrt{3}}{2}}$$

f) $g(-\theta)$

$$= \cos(-30^\circ) = \cos 30^\circ = \boxed{\frac{\sqrt{3}}{2}}$$

↑
even function
 $f(-x) = f(x)$