Section 4.3 Right Triangle Trigonometry

Objective: In this lesson you learned how to evaluate trigonometric functions of acute angles and how to use the fundamental trigonometric identities.

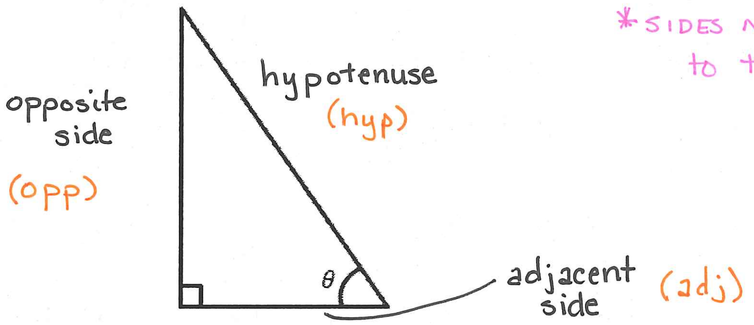
Important Vocabulary		
Hypotenuse	**Opposite Side**	**Adjacent Side**
Angle of Elevation	**Angle of Depression**	

I. The Six Trigonometric Functions

What you should learn:

How to evaluate trigonometric functions of acute angles

In the right triangle below, label the three sides of the triangle relative to the angle labeled θ as (a) the **hypotenuse**, (b) the **opposite side**, and (c) the **adjacent side**.



*SIDES named in relation to the angle θ *

Let θ be an acute angle of a right triangle. Define the six trigonometric functions of the angle θ using opp = the length of the side opposite θ, adj = the length of the side adjacent to θ, and hyp = the length of the hypotenuse.

$\sin\theta =$ opp/hyp $\cos\theta =$ adj/hyp

$\tan\theta =$ opp/adj $\csc\theta =$ hyp/opp

$\sec\theta =$ hyp/adj $\cot\theta =$ adj/opp

The cosecant function is the reciprocal of the sine function. The cotangent function is the reciprocal of the tangent function. The secant function is the reciprocal of the cosine function.

Give the sines, cosines, and tangents of the following special angles:

$\sin 30° = \sin\frac{\pi}{6} =$ $1/2$

$\cos 30° = \cos\frac{\pi}{6} =$ $\sqrt{3}/2$

$\tan 30° = \tan\frac{\pi}{6} =$ $\sqrt{3}/3$

$\sin 45° = \sin\frac{\pi}{4} =$ $\sqrt{2}/2$

$\cos 45° = \cos\frac{\pi}{4} =$ $\sqrt{2}/2$

$\tan 45° = \tan\frac{\pi}{4} =$ 1

$\sin 60° = \sin\frac{\pi}{3} =$ $\sqrt{3}/2$

$\cos 60° = \cos\frac{\pi}{3} =$ $1/2$

$\tan 60° = \tan\frac{\pi}{3} =$ $\sqrt{3}$

Cofunctions of complementary angles are equal. If θ is an acute angle, then:

$\sin(90° - \theta) =$ $\cos\theta$

$\cos(90° - \theta) =$ $\sin\theta$

$\tan(90° - \theta) =$ $\cot\theta$

$\cot(90° - \theta) =$ $\tan\theta$

$\sec(90° - \theta) =$ $\csc\theta$

$\csc(90° - \theta) =$ $\sec\theta$

II. Trigonometric Identities

What you should learn:

How to use the fundamental trigonometric identities

List six reciprocal identities:

1) $\sin\theta = \frac{1}{\csc\theta}$

2) $\cos\theta = \frac{1}{\sec\theta}$

3) $\tan\theta = \frac{1}{\cot\theta}$

4) $\csc\theta = \frac{1}{\sin\theta}$

5) $\sec\theta = \frac{1}{\cos\theta}$

6) $\cot\theta = \frac{1}{\tan\theta}$

List two quotient identities:

1) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

2) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

List three Pythagorean identities:

1) $\sin^2\theta + \cos^2\theta = 1$

2) $\tan^2\theta + 1 = \sec^2\theta$

3) $1 + \cot^2\theta = \csc^2\theta$

note: $\sin^2\theta = [\sin\theta]^2$

III. Applications Involving Right Triangles

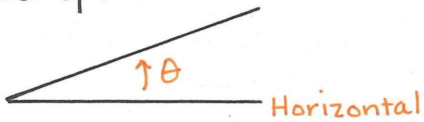
What does it mean to "solve a right triangle?"

To find/calculate all angle measures and side lengths of a right triangle.

> What you should learn:
>
> How to use trigonometric functions to model and solve real-life problems

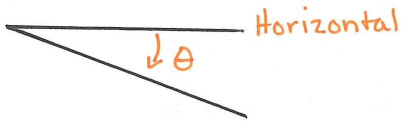
An **angle of elevation** is:

the angle from the horizontal upward to an object.



An **angle of depression** is:

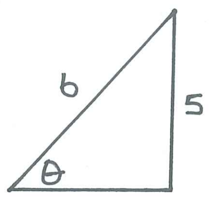
the angle from the horizontal downward to an object



Section 4.3 Examples – Right Triangle Trigonometry

(1) Sketch a right triangle corresponding to the trigonometric function of the acute angle θ.

$$\sin\theta = \frac{5}{6}$$ ← opp. ← hyp.



(2) Use the given function value(s) and the trigonometric identities to find the indicated trigonometric functions.

$$\sin 60° = \frac{\sqrt{3}}{2}, \qquad \cos 60° = \frac{1}{2}$$

a. $\tan 60°$

$$= \frac{\sin 60°}{\cos 60°} = \frac{\sqrt{3}/2}{1/2} = \boxed{\sqrt{3}}$$

b. $\sin 30°$

$$= \sin(90° - 60°) = \cos 60° = \boxed{\frac{1}{2}}$$

COFUNCTIONS

c. $\cos 30°$

$$= \cos(90° - 60°) = \sin 60° = \boxed{\frac{\sqrt{3}}{2}}$$

Cofunctions

d. $\cot 60°$

$$= \frac{\cos 60°}{\sin 60°} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

(3) Use identities to transform one side of the equation into the other $\left(0 < \theta < \frac{\pi}{2}\right)$.

$\tan\theta \cot\theta = 1$

$$\frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} =$$

$$1 = 1 \checkmark$$