Section 4.1 Radian and Degree Measure

Objective: In this lesson you learned how to describe an angle and to convert between degree and radian measure

Important Vocabulary			
Degree	**Angle**	**Initial Side**	**Terminal Side**
Standard Position	**Positive Angle**	**Negative Angle**	**Coterminal**
Radian	**Central angle of a circle**	**Complementary Angles**	**Supplementary Angles**

What you should learn:

How to describe angles

I. Angles

An **angle** is determined by:

ROTATING A RAY (a half-line) ABOUT its endpoint

The **initial side** of an angle is:

the starting position of the rotated ray.

The **terminal side** of an angle is:

the position of the ray after rotation

The **vertex** of an angle is:

where the terminal and initial sides of an angle meet.

An angle is in **standard position** when: the vertex is at the origin of a coordinate system, and its initial side is on the positive x-axis

A **positive angle** is generated by a(n) counterclockwise rotation; whereas a **negative angle** is generated by a(n) clockwise rotation.

If two angles are **coterminal**, then they have:

the same initial side and terminal side

II. Radian Measure

What you should learn:

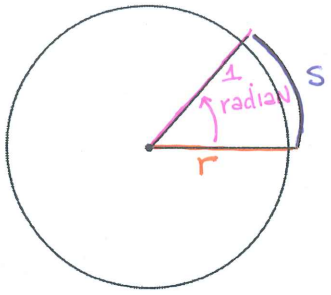
How to use radian measure

The measure of an angle is determined by:

the amount of rotation from the initial side to the terminal side

One **radian** is the measure of a central angle θ that:

intercepts an arc s equal in length to the radius r of the circle



Algebraically this means that $\theta = \frac{s}{r}$, where θ is measured in radians

Greek Letter "theta"

A **central angle of one full revolution** (counterclockwise) corresponds to an arc length of

$s =$ $2\pi r$.

The radian measure of an angle one full revolution is 2π radians. A half revolution corresponds to an angle of π radians. Similarly $\frac{1}{4}$ revolution corresponds to an angle of $\pi/2$ radians, and $\frac{1}{6}$ revolution corresponds to an angle of $\pi/3$ radians.

Angles with measures between 0 and $\frac{\pi}{2}$ radians are ACUTE angles. Angles with measures between $\frac{\pi}{2}$ and π radians are OBTUSE angles.

III. Degree Measure

What you should learn:

How to use degree measure and convert between degrees and radian measure

A full revolution (counterclockwise) around a circle corresponds to 360 degrees. A half revolution around a circle corresponds to 180 degrees.

To convert degrees to radians, you:

$$\times \frac{\pi \text{ rad.}}{180^\circ}$$

To convert radians to degrees, you:

$$\times \frac{180^\circ}{\pi \text{ rad.}}$$

IV. Linear and Angular Speed

What you should learn:
How to use angles to model and solve real-life problems

For a circle of radius r, a central angle θ intercepts an arc f

length s given by $s = r\theta$ where θ is measured in radians.

Note that if $r = 1$, then $s = \theta$, and the radian measure of θ

equals the arc length.

Consider a particle moving at a constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed** of the particle is

$$\text{linear speed} = \text{arc length}/\text{time} = s/t$$

If θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** of the particle is

$$\text{angular speed} = \text{central angle}/\text{time} = \theta/t$$

Section 4.1 Examples – Radian and Degree Measure

(1) Determine the quadrant in which the angle lies.

a) $55°$ Quad I

b) $215°$ III

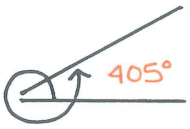
c) $\frac{\pi}{6}$ I

d) $\frac{5\pi}{4}$ III

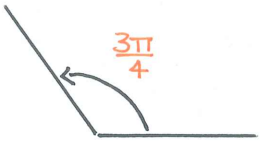
(2) Sketch the angle in standard position.

a) $45°$

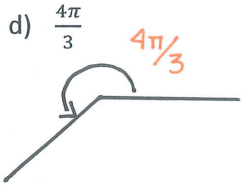
b) $405°$



c) $\frac{3\pi}{4}$



d) $\frac{4\pi}{3}$



(3) Determine two coterminal angles (one positive and one negative) for the given angle.

$\theta = 35°$

$35° + 360° = 395°$

$35° - 360° = -325°$

(4) Convert the angle from degrees to radians.

NOTE: radian measure does not have a unit label

a) $75°$

$75° \times \frac{\pi \text{ rad.}}{180°} = \frac{15\pi}{36}$

b) $-45°$

$-45° \times \frac{\pi \text{ rad.}}{180°} = -\frac{\pi}{4}$

(5) Convert the angle from radians to degrees.

a) $\frac{2\pi}{3}$

MUST included the label!

$\frac{2\pi}{3} \times \frac{180°}{\pi} = 120°$

b) $\frac{3\pi}{2}$

$\frac{3\pi}{2} \times \frac{180°}{\pi} = 270°$

(6) Find the length of the arc on a circle of radius r intercepted by a central angle θ.

$r = 14$ inches, $\theta = 180°$

$\theta = 180° = \pi$

$s = r\theta = \boxed{14\pi}$