Section 3.3 Properties of Logarithms

Objective: In this lesson you learned how to rewrite logarithmic functions with different bases and how to use properties of logarithms to evaluate, rewrite, expand, or condense logarithmic expressions.

I. Change of Base

What you should learn:

How to rewrite logarithms with different bases

Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. The **change-of-base formula** states that:

$\log_a x$ CAN BE CONVERTED TO A DIFFERENT BASE USING ANY ONE OF THE FOLLOWING:

BASE b: $\log_a x = \frac{\log_b x}{\log_b a}$

BASE e: $\log_a x = \frac{\log_e x}{\log_e a} = \frac{\ln x}{\ln a}$

BASE 10: $\log_a x = \frac{\log x}{\log a}$

Explain how to use a calculator to evaluate $\log_8 20$.

$$\log_8 20 = \frac{\log 20}{\log 8} = \frac{\ln 20}{\ln 8}$$

II. Properties of Logarithms

What you should learn:

How to use properties of logarithms to evaluate or rewrite logarithmic expressions

Let a be a positive number such that $a \neq 1$; let n be a real number; and let u and v be positive real numbers.

Complete the following logarithm properties:

1) $\log_a(uv) =$ $\log_a u + \log_a v$

2) $\log_a \frac{u}{v} =$ $\log_a u - \log_a v$

3) $\log_a u^n =$ $n \log_a u$

III. Rewriting Logarithmic Expressions

What you should learn:

How to use properties of logarithms to expand or condense logarithmic expressions

To expand a logarithmic expression means to:

USE PROPERTIES OF LOGARITHMS TO WRITE LOGS AS SUMS, PRODUCTS, AND DIFFERENCES

To condense a logarithmic expression means to:

TO WRITE A LOG FUNCTION AS A SINGLE LOGARITHM USE THE PROPERTIES OF LOGARITHMS.

IV. Applications of Properties of Logarithms

What you should learn:

How to use properties of logarithmic functions to model and solve real-life problems

One way of finding a model for a set of nonlinear data is to take the natural log of each of the x-values and y-values of the data set. If the points are graphed and fall on a straight line, then the x-values and y-values are related by the equation $\ln y = m \ln x$, where m is the slope of the straight line.

Section 3.3 Examples – Properties of Logarithms

(1) Rewrite the logarithm as a ratio of (a) common logarithms and (b) natural logarithms.

$$\log_5 x$$

a) $\log_5 x = \dfrac{\log x}{\log 5}$

B) $\log_5 x = \dfrac{\ln x}{\ln 5}$

(2) Use the properties of logarithms to rewrite and simplify the logarithmic expression.

$$\log_2 4^2 \cdot 3^4$$

$= \log_2 2^4 \cdot 27$ ← 3^4 IS NOT INCLUDED AS PART OF THE log!

$= 4 \cdot 27$

$\boxed{= 108}$

(3) Use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms.

$$\ln \frac{xy}{z}$$

$= \ln xy - \ln z$

$\boxed{= \ln x + \ln y - \ln z}$

(4) Condense the expression to the logarithm of a single quantity.

$$3 \log x + 2 \log y - 4 \log z$$

$= \log x^3 + \log y^2 - \log z^4$

$= \log x^3 y^2 - \log z^4$

$\boxed{= \log \dfrac{x^3 y^2}{z^4}}$ ← COULD PUT [ ] AROUND THE FRACTION IF YOU WANTED.