

## Chapter 3 Averages and Variation

### Section 3.1 Measures of Central Tendency: Mode, Median, and Mean

Objective: In this lesson you learned how to compute, interpret, and explain mean, median, and mode.

Important Vocabulary					
Average	Mode	Median	Mean	Trimmed Mean	Weighted Average
Important Notation					
$\Sigma x$	$\bar{x}$	$\mu$			

What is an average used to describe?

*a single number used to describe the entire sample or population.*

#### I. Mode

The **mode**: of a data set is the value that occurs most frequently.

What are some disadvantages to using mode?

- Not every data set has a mode.
- Not very stable; one number can change everything

What are some advantages to using mode?

- Useful for most frequent data value

Focus Points:

- Compute mean, median, and mode from raw data
- Interpret what mean, median, and mode tell you
- Explain how mean, median, and mode can be affected by extreme data values
- What is a trimmed mean? How do you compute it?

#### II. Median

The **median**: is the central value of an ordered distribution

How to find the median

1. Order the data  
smallest  $\rightarrow$  largest.
2. For an odd number of data values  
Median = middle data value.
3. For an even number of data values  
Median =  $\frac{\text{sum of two middle values}}{2}$  *essentially average them.*

What are some advantages to using median?

- Median is based on position, rather than specific values.
- If extreme values change, median usually does not.

### III. Mean

The mean: uses the exact value of each entry.

What does the following notation mean and when is it used?

- $\sum x$  - sum of data values  
"Sigma x"
- $\bar{x}$  - symbol for the mean of a sample.  
"x-bar"
- $\mu$  - symbol for the mean of a population  
"greek letter mu; pronounced "mew"

How to find the mean

1. Compute  $\sum x$

Sample ( $\bar{x}$ )  
$$\bar{x} = \frac{\sum x}{n}$$

Population ( $\mu$ )  
$$\mu = \frac{\sum x}{N}$$

2. Divide  $\sum x$  by  
the number of  
data values

$n$  = # in  
sample

$N$  = # in  
population

What do averages tell us?

Averages provide a one-number summary of a data set

- Mode - gives us the data value(s) that occur most frequently
- Median - middle value of a data set that is in increasing order
- Mean - Average of ALL the data values.

What is a major disadvantage to using mean?

Can be affected by extreme values  
(outliers)

What is a trimmed mean?

the average of the data after "trimming" a specified percent from the data set.

Removing

How to compute a 5% trimmed mean

1. order the data
2. Delete the top/Bottom 5% of the data
3. Compute the mean of the remaining 90%

In general, when a data distribution is mound-shaped symmetrical, the values for mean, median, and mode are:

*approximately the same.*

#### IV. Weighted Average

$$\text{Weighted Average} = \frac{\sum x \cdot \omega}{\sum \omega}$$

$x = \text{data values}$       $\omega = \text{weight}$

Focus Point:

- Compute a weighted average

### Section 3.1 Examples – Measures of Central Tendency: Mode, Median, Mean

(1) Belleview College must make a report to the budget committee about the average credit hour load a full-time student carries. (A 12-hour credit load is the minimum requirement for full-time students. For the same tuition, students may take up to 20 credit hours.) A random sample of 40 students yielded the following information (in credit hours):

17 12 14 17 13 16 18 20 13 12  
 12 17 16 15 14 12 12 13 17 14  
 15 12 15 16 12 18 20 19 12 15  
 18 14 16 17 15 19 12 13 12 15

a. Organize the data from smallest to largest number of credit hours.

12 12 12 12 12 12 12 12 12 12  
13 13 13 13 14 14 14 14 15 15  
15 15 15 15 16 16 16 16 17 17  
17 17 17 18 18 18 19 19 20 20

b. Since there are a(n) even (odd, even) number of values, we add the two middle values and divide by 2 to get the median. What is the median credit hour load?

$$Median = \frac{15+15}{2} = 15$$

c. What is the mode of this distribution? Is it different from the median?

mode = 12      yes it's different from the median

d. If the budget committee is going to fund the college according to the average student credit hour load (more money for higher loads), which of these two averages do you think the college will report?

Since the median is higher, the college will probably report it and indicate that the average being used is median.

(2) Barron's Profiles of American Colleges, 19<sup>th</sup> Edition, lists average class size for introductory lecture courses at each of the profiled institutions. A sample of 20 colleges and universities in California showed class sizes for introductory lecture courses to be

<del>14</del>	20	20	20	20	23	25	30	30	30
35	35	35	40	40	42	50	50	80	<del>80</del>

a. Compute the mean for the entire sample.

$$\bar{x} = \frac{\sum x}{n} = \frac{719}{20} \approx 36.0 \quad \leftarrow \text{no decimal because data is integers}$$

b. Compute a 5% trimmed mean for the sample.

$$20 \times 5\% = 1$$

$$\bar{x}_{5\%} = \frac{\sum x}{n} = \frac{625}{18} \approx 34.7 \approx 35$$

c. Find the median of the original data set.

$$\text{Median} = \frac{30+35}{2} = 32.5 \quad \leftarrow \text{Always keep decimal values for median!}$$

d. Find the median of the 5% trimmed data set. Does the median change when you trim the data?

$$\text{Median}_{5\%} = 32.5$$

Trimming the same number of values from the top & bottom leaves the middle position unchanged.

e. Is the trimmed mean or the original mean closer to the median?

The trimmed mean is closer to the median.

(3) Suppose your midterm test score is 83 and your final exam score is 95. Using weights of 40% for the midterm and 60% for the final exam, compute the weighted average of your scores. If the minimum average for an A is 90, will you earn an A?

$$\frac{83(.4) + 95(.6)}{(.4 + .6)} = 90.2$$

Your average is high enough to earn an A.