Section 2.6 Rational Functions and Asymptotes

Objective: In this lesson you learned how to determine the domains and find asymptotes of rational functions

Important Vocabulary		
Rational function	Vertical asymptotes	Horizontal asymptotes

I. Introduction to Rational Functions

What you should learn:
How to find the domains of rational functions

The domain of a rational function of x incudes all real numbers

except: X-VALUES THAT MAKE THE DENOMINATOR EQUAL ZERO

To find the domain of a rational function of x, you:

SET THE DENOMINATOR EQUAL TO ZERO, AND SOLVE FOR X.

II. Horizontal and Vertical Asymptotes

What you should learn:
How to find horizontal and vertical asymptotes of graphs of rational functions

The notation "$f(x) \to 5$ as $x \to \infty$" means:

as x INCREASES, f(x) APPROACHES 5

COMMON NOTATION USED IN CALCULUS FOR LIMITS

Let f be the rational function given by

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$$

where $N(x)$ and $D(x)$ have no common factors.

1) The graph of f has vertical asymptotes at

THE ZEROS OF D(X).

2) The graph of f has at most one horizontal asymptote determined by

COMPARING THE DEGREES OF N(X) AND D(X).

a. If $n < m$, the LINE $\boxed{Y=0}$ IS A HORIZONTAL ASYMPTOTE

b. If $n = m$, the LINE $\boxed{Y = a_n / b_n}$ IS A HORIZONTAL ASYMPTOTE

c. If $n > m$, the GRAPH HAS NO HORIZONTAL ASYMPTOTE

What makes a hole in the graph of a rational function?

A SINGLE EXCLUDED VALUE.

$$f(x) = \frac{(x+1)\cancel{(x-2)}}{\cancel{(x-2)}(x+3)} = \frac{x+1}{x+3},\ x \neq 2, -3$$

FACTORED OUT OR CANCELLED TERMS CREATE HOLES IN THE GRAPH.

Section 2.7 Graphs of Rational Functions

Objective: In this lesson you learned how to sketch graphs of rational functions

Important Vocabulary

Slant (or oblique) asymptote

I. The Graph of a Rational Function

What you should learn:

How to analyze and sketch graphs of rational functions

To sketch the graph of the rational function

$f(x) = N(x)/D(x)$, where $N(x)$ and $D(x)$ are polynomials

with no common factors, you:

1. Simplify f, if possible (Holes in graph?)
2. Find & Plot y-intercept (x=0)
3. Find zeros of numerator (x-intercepts)
4. Find zeros of denominator (Vertical Asymptote(s))
5. Find and sketch any other Asymptotes (Horizontal/Slant)
6. Plot atleast one point between and one point beyond each x-intercept
7. Use smooth curves to complete the graph

II. Slant Asymptotes

What you should learn:

How to sketch graphs of rational functions that have slant asymptotes

To find the equation of a slant asymptote, you:

Use polynomial long division

$$f(x) = \frac{x^2 - x}{x+1} = \boxed{x-2} + \frac{2}{x-1}$$

Quotient gives the slant asymptote equation

$y = x - 2$

Section 2.6/2.7 Examples – Rational Functions

1) Graph the function (a) by finding the intercepts, (b) finding the asymptotes, and (c) using an xy-chart to fill in the rest.

$f(x) = -\frac{2}{x+3}$

A) Y-INT $(x=0)$

$y = {}^{-2}/_{3}$

X-INT $(y=0)$

NONE

B) VERTICAL Asym.

$x+3=0$

$\boxed{x=-3}$

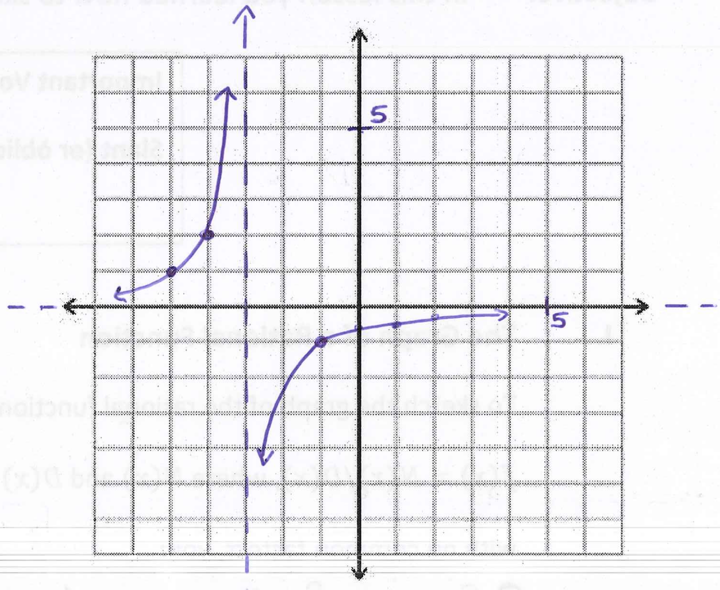
HORIZONTAL Asym.

$x^1 > 2x^0 \leftarrow n<m$ CONDITION

$y=0$

C)

x	y
-5	1
-1	-1
1	$-\frac{1}{2}$
3	$-\frac{1}{3}$



2) Graph the function (a) by finding the intercepts, (b) finding the asymptotes, and (c) using an xy-chart to fill in the rest.

$f(x) = \frac{2x^2-5x+5}{x-2}$

A) Y-INT $(x=0)$

$y = {}^{-5}/_{2}$

X-INT $(y=0)$

$2x^2-5x+5=0$

NONE

B) VERTICAL Asym.

$x-2=0$

$\boxed{x=2}$

HORIZONTAL Asym.

$x^2 > x \leftarrow n>m$ CONDITION

NONE

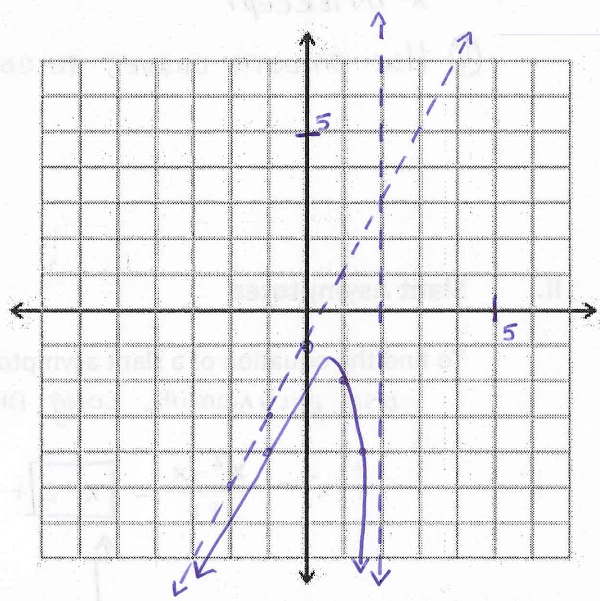
SLANT Asym.

$\boxed{2x-1}$ ← SLANT $y=2x-1$

$$\begin{array}{r} 2x-1 \\ x-2\,\overline{)\,2x^2-5x+5} \\ -(2x^2-4x) \\ -x+5 \\ -(-x+2) \\ 3 \end{array}$$

C)

x	y
1	-2
1.5	-4
-1	-4
-2	



NOTE:
- WILL NEVER HAVE BOTH AT THE SAME TIME!
- SLANT ONLY HAPPENS WHEN $D(x) < N(x)$ by EXACTLY 1.