Section 2.5 The Fundamental Theorem of Algebra

Objective: In this lesson you learned how to determine the numbers of zeros of polynomial functions and find them

Important Vocabulary		
Fundamental Theorem of Algebra	Linear Factorization Theorem	Conjugates

I. The Fundamental Theorem of Algebra

In the complex numbers system, every nth-degree polynomial function has PRECISELY n zeros. An nth-degree polynomial can be factored into PRECISELY n linear factors.

What you should learn:

How to use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function and find all zeros of polynomial functions, including complex zeros

II. Conjugate Pairs

Let $f(x)$ be a polynomial function that has real coefficients. If $a + bi$, where $b \neq 0$, is a zero of the function, then we know that $a - bi$ is also a zero of the function.

What you should learn:

How to find conjugate pairs of complex zeros

III. Factoring a Polynomial

To write a polynomial of degree $n > 0$ with real coefficients as a product without complex factors, write the polynomial as:

THE PRODUCT OF LINEAR OR QUADRATIC FACTORS, WHERE THE QUADRATIC FACTORS HAVE NO REAL ZEROS

What you should learn:

How to find zeros of polynomials by factoring

A quadratic factor with no real zeros is said to be PRIME.

Section 2.5 Examples – The Fundamental Theorem of Algebra

(1) For the following problem, (a) find ALL the zeros of the function, and (b) write the polynomial as a product of linear factors. You might have to use the **Quadratic Formula** if you can't factor.

$$f(y) = 81y^4 - 625$$

A) $81y^4 - 625 = 0$

$(9y^2 - 25)(9y^2 + 25) = 0$

$y^2 = \frac{25}{9}$ $y^2 = \frac{-25}{9}$

$\boxed{y = \pm \frac{5}{3}}$ $\boxed{y = \pm \frac{5}{3}i}$

B) $f(y) = (x - \frac{5}{3})(x + 5/3)(x - \frac{5}{3}i)(x + \frac{5}{3}i)$

$= (3x-5)(3x+5)(3x-5i)(3x+5i)$

(2) Find a fourth degree polynomial that has 2, 2, and $4 - i$ as zeros.

COMPLEX SOLUTIONS ALWAYS COME IN PAIRS!

$P(x) = (x-2)^2[x-(4-i)][x-(4+i)]$

$= (x^2-4x+4)[x^2 - x(4+i) - x(4-i) + (4-i)(4+i)]$

$= (x^2-4x+4)[x^2 - 4x - \cancel{ix} - 4x + \cancel{ix} + 16 + \cancel{4i} - \cancel{4i} - i^2]$

$= (x^2-4x+4)[x^2 - 8x + 17]$

$= x^4 - 8x^3 + 17x^2 - 4x^3 + 32x^2 - 68x + 4x^2 - 32x + 68$

$\boxed{= x^4 - 12x^3 + 53x^2 - 100x + 68}$