

Irrational and Imaginary Root Theorems

State the number of complex zeros and the possible number of real and imaginary zeros for each function.

1) $f(x) = x^2 + 6x - 38$

2) $f(x) = x^4 - 9x^2 + 18$

3) $f(x) = 5x^5 + 36x^3 + 7x$

4) $f(x) = 3x^3 + 2x^2 - x$

5) $f(x) = 27x^6 + 208x^3 - 64$

6) $f(x) = 27x^9 + 8x^6 - 27x^3 - 8$

A polynomial function with rational coefficients has the follow zeros. Find all additional zeros.

7) $-5, i$

8) $-1 + i, \sqrt{5}$

9) $-3 + \sqrt{5}, -i$

10) $2, -2 + \sqrt{10}$

11) $-1, 5, -2 + \sqrt{5}$

12) $2 - 2i, 1 - 2i, 1 + 2i$

13) $2 - 3i$ mult. 2

14) $-\frac{5}{2}, \sqrt{5}$ mult. 2

Write a polynomial function of least degree with integral coefficients that has the given zeros.

15) 0, 2, $\sqrt{3}$

16) -5, $\sqrt{3}$

17) -1, $2i$

18) $2i, -2i, 2 + 2i$

19) $-2i, 2 + 2\sqrt{2}$

20) $\sqrt{6}, -3 + \sqrt{5}$

Critical thinking questions:

21) Write a polynomial function of fifth degree with integral coefficients that has $2i$ as a zero.

22) True or False: A polynomial function of third degree with integral coefficients can have 2 and $2i$ as zeros.

Irrational and Imaginary Root Theorems

State the number of complex zeros and the possible number of real and imaginary zeros for each function.

1) $f(x) = x^2 + 6x - 38$

of complex zeros: 2

Possible # of real zeros: 2 or 0

Possible # of imaginary zeros: 2 or 0

2) $f(x) = x^4 - 9x^2 + 18$

of complex zeros: 4

Possible # of real zeros: 4, 2, or 0

Possible # of imaginary zeros: 4, 2, or 0

3) $f(x) = 5x^5 + 36x^3 + 7x$

of complex zeros: 5

Possible # of real zeros: 5, 3, or 1

Possible # of imaginary zeros: 4, 2, or 0

4) $f(x) = 3x^3 + 2x^2 - x$

of complex zeros: 3

Possible # of real zeros: 3 or 1

Possible # of imaginary zeros: 2 or 0

5) $f(x) = 27x^6 + 208x^3 - 64$

of complex zeros: 6

Possible # of real zeros: 6, 4, 2, or 0

Possible # of imaginary zeros: 6, 4, 2, or 0

6) $f(x) = 27x^9 + 8x^6 - 27x^3 - 8$

of complex zeros: 9

Possible # of real zeros: 9, 7, 5, 3, or 1

Possible # of imaginary zeros: 8, 6, 4, 2, or 0

A polynomial function with rational coefficients has the follow zeros. Find all additional zeros.

7) $-5, i$

 $-i$

8) $-1 + i, \sqrt{5}$

 $-1 - i, -\sqrt{5}$

9) $-3 + \sqrt{5}, -i$

 $-3 - \sqrt{5}, i$

10) $2, -2 + \sqrt{10}$

 $-2 - \sqrt{10}$

11) $-1, 5, -2 + \sqrt{5}$

 $-2 - \sqrt{5}$

12) $2 - 2i, 1 - 2i, 1 + 2i$

 $2 + 2i$

13) $2 - 3i$ mult. 2
 $2 + 3i$ mult. 2

14) $-\frac{5}{2}, \sqrt{5}$ mult. 2
 $-\sqrt{5}$ mult. 2

Write a polynomial function of least degree with integral coefficients that has the given zeros.

15) 0, 2, $\sqrt{3}$
 $f(x) = x^4 - 2x^3 - 3x^2 + 6x$

16) -5, $\sqrt{3}$
 $f(x) = x^3 + 5x^2 - 3x - 15$

17) -1, $2i$
 $f(x) = x^3 + x^2 + 4x + 4$

18) $2i, -2i, 2 + 2i$
 $f(x) = x^4 - 4x^3 + 12x^2 - 16x + 32$

19) $-2i, 2 + 2\sqrt{2}$
 $f(x) = x^4 - 4x^3 - 16x - 16$

20) $\sqrt{6}, -3 + \sqrt{5}$
 $f(x) = x^4 + 6x^3 - 2x^2 - 36x - 24$

Critical thinking questions:

21) Write a polynomial function of fifth degree with integral coefficients that has $2i$ as a zero.
Many answers. Ex: $f(x) = x^5 + 4x^3$

22) True or False: A polynomial function of third degree with integral coefficients can have 2 and $2i$ as zeros.
True