Section 2.4 Complex Numbers

Objective: In this lesson you learned how to perform operations with complex numbers

Important Vocabulary		
Complex Number	Complex conjugates	Imaginary axis
Real axis	Bounded	Unbounded

I. The Imaginary Unit i

What you should learn:

How to use the imaginary unit i to write complex numbers

Mathematicians created an expanded system of numbers using the **imaginary unit i**, defined as $i =$ $\sqrt{-1}$,

because:

MATHEMATICIANS WANTED A WAY TO SOLVE PROBLEMS LIKE $X^2 = -1$.

By definition, $i^2 =$ -1.

For the complex number $a + bi$, if $b = 0$, the number $a + bi = a$ is a(n) REAL NUMBER. If $b \neq 0$, the number $a + bi$ is a(n) IMAGINARY NUMBER. If $a = 0$, the number $a + bi = bi$, where $b \neq 0$, is called a(n) PURE IMAGINARY NUMBER. The set of complex numbers consists of the set of REAL NUMBERS and the set of IMAGINARY NUMBERS.

Two complex numbers $a + bi$ and $c + di$, written in standard form, are equal to each other if:

$a = c$ AND $B = D$.

II. Operations with Complex Numbers

What you should learn:

How to add, subtract, and multiply complex numbers

To add two complex numbers, you:

$(a + Bi) + (c + di) = (a + c) + (b + d)i$

To subtract two complex numbers, you:

$(a + Bi) - (c + di) = (a - c) + (b - di)$

The additive identity in the complex number system is 0 (ZERO).

The additive inverse of the complex number $a + bi$ is $-a - bi$.

To multiply two complex numbers $a + bi$ and $c + di$, you:

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III. Complex Conjugates

What you should learn:

How to use complex conjugates to write the quotient of two complex numbers in standard form

The product of a pair of complex conjugates is a(n) REAL number.

To find the quotient of the complex numbers $a + bi$ and $c + di$, where c and d are not both zero, you:

$$\frac{a+Bi}{c+Di} \times \frac{c-Di}{c-Di} =$$

MULTIPLY THE NUMERATOR & DENOMINATOR BY THE COMPLEX CONJUGATE OF $C+Di$

Section 2.4 Examples – Complex Numbers

(1) Perform the indicated operation. Write the answer in standard form.

a) $(3 - i) + (2 + 3i)$

$\boxed{5 + 2i}$

b) $3 - (-2 + 3i) + (-5 + i)$

$3 + 2 - 3i - 5 + i$

$\boxed{-2i}$

c) $(2 - i)(4 + 3i)$

$8 + 6i - 4i - 3i^2$

$\boxed{11 + 2i}$

REMEMBER:

i

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

d) $\frac{2+3i}{4-2i} \times \frac{4+2i}{4+2i}$

$$\frac{8 + 4i + 12i + 6i^2}{16 + 8i - 8i - 4i^2} = \frac{2 + 16i}{20} = \boxed{\frac{1 + 8i}{10}}$$

(2) Solve the quadratic equation.

a) $x^2 + 4 = 0$

$x^2 = -4$

$\boxed{x = \pm 2i}$

b) $3x^2 - 2x + 5 = 0$

$$x = \frac{2 \pm \sqrt{4 - 60}}{6}$$

$$= \frac{2 \pm \sqrt{-56}}{6}$$

$$= \frac{-2 \pm 2i\sqrt{14}}{6} = \boxed{\frac{-1 \pm i\sqrt{14}}{3}}$$