Section 2.3 Real Zeros of Polynomial Functions

Objective: In this lesson you learned how to use long division and synthetic division to divide polynomials by other polynomials and how to find the rational and real zeros of polynomial functions

Important Vocabulary		
Long division of polynomials	**Division Algorithm**	**Synthetic division**
Remainder Theorem	**Factor Theorem**	**Upper bound**
Lower bound		

I. Long Division of Polynomials

What you should learn:

How to use long division to divide polynomials by other polynomials

When dividing a polynomial $f(x)$ by another polynomial $d(x)$, if the remainder $r(x) = 0$, $d(x)$ divides evenly into $f(x)$.

If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$\underbrace{f(x)}_{\text{Dividend}} = \underbrace{d(x)}_{\text{DIVISOR}}\underbrace{q(x)}_{\text{QUOTIENT}} + \underbrace{r(x)}_{\text{REMAINDER}}$$

where:

$r(x)=0$ or the degree of $r(x)$ is less than the degree of $d(x)$.

The rational expression $f(x)/d(x)$ is **improper** if:

the degree of $f(x) \geq$ the degree of $d(x)$

The rational expression $r(x)/d(x)$ is **proper** if:

the degree of $r(x) <$ the degree of $d(x)$

Before applying the Division Algorithm, you should:

① Write the dividend and divisor in decreasing powers

② Insert placeholders for missing powers

II. Synthetic Division

Can synthetic division be used to divide a polynomial by $x^2 - 5$? Explain.

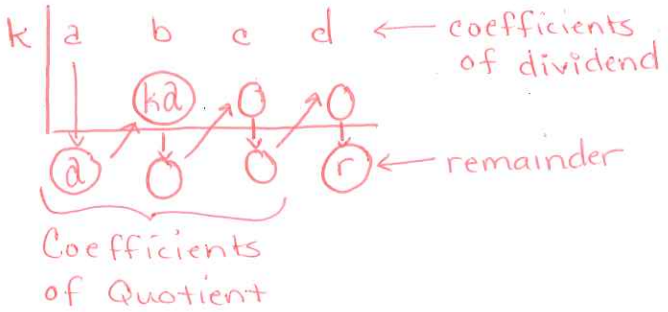
no, because x^2

What you should learn:
How to use synthetic division to divide polynomials by binomials of the form $(x - k)$

Can synthetic division be used to divide a polynomial by $x + 4$? Explain.

yes, $x+4$ fits the "pattern"

To divide $ax^3 + bx^2 + cx + d$ by $x - k$, use the following pattern:



III. The Remainder and Factor Theorems

What you should learn:
How to use the Remainder and Factor Theroems

To use the Remainder Theorem to evaluate a polynomial function $f(x)$ at $x = k$, you:

use synthetic division to divide $f(x)$ by $x-k$. The remainder will be $f(k)$.

To use the Factor Theorem to show that $(x - k)$ is a factor of a polynomial function $f(x)$, you:

use synthetic division on f(x) with the factor x-k. If the remainder is 0, the x-k is a factor.

List three facts about the remainder r, obtained in the synthetic division of $f(x)$ by $x - k$:

1) the remainder r gives the value of f at x=k. $r = f(k)$

2) If $r = 0$, then $(x-k)$ is a factor of $f(x)$.

3) If $r = 0$, then $(k,0)$ is an x-intercept of the graph of f.

IV. The Rational Zero Test

> **What you should learn:**
>
> How to use the Rational Zero Test to determine possible rational zeros of polynomial functions

Describe the purpose of the Rational Zero Test:

to find possible rational zeros of a polynomial function.

State the **Rational Zero Test**:

If a polynomial f(x) has integer coefficients, then every rational zero of f has the form

$$\text{Rational Zero} = \frac{P}{Q} \begin{array}{l} \leftarrow \text{factors of constant} \\ \leftarrow \text{factors of leading coefficient} \end{array}$$

To use the Rational Zero Test, you:

1. List all rational factors of p and Q.
2. trial and error to find zeros of the polynomial

V. Other Tests for Zeros of Polynomials

> **What you should learn:**
>
> How to use Descartes's Rule of Signs and the Upper and Lower Bound Rules to find zeros of polynomials

State Descartes's Rule of Signs:

Let f(x) be a polynomial w/ real coefficients:

1. Positive Real Zeros – ≤ to number of sign Δs (decrease by 2)
2. Negative Real Zeros – ≤ to number of sign Δs in f(-x) (decrease by 2)

Section 2.3 Examples – Real Zeros of Polynomial Functions

(1) Use long division to divide.

$$(x + 8 + 6x^3 + 10x^2) \div (2x^2 + 1)$$

$$\begin{array}{r|l} & 3x + 5 \\ 2x^2 + 0x + 1 & 6x^3 + 10x^2 + x + 8 \\ & -(6x^3 + 0x^2 + 3x) \\ & \quad 10x^2 - 2x + 8 \\ & \quad -(10x^2 + 0x + 5) \\ & \qquad -2x + 3 \end{array}$$

$$3x + 5 + \frac{-2x+3}{2x^2+1}$$

(2) Use synthetic division to divide.

$$(3x^3 - 17x^2 + 15x - 25) \div (x - 5)$$

$$\begin{array}{r|rrrr} 5 & 3 & -17 & 15 & -25 \\ & \downarrow & 15 & -10 & 25 \\ \hline & 3 & -2 & 5 & 0 \end{array}$$

$$3x^2 - 2x + 5$$

(3) Use the Remainder Theorem and Synthetic Division to evaluate the function at each given value.

$$g(x) = 2x^6 + 3x^4 - x^2 + 3$$

a) $g(2)$ $= 175$ b) $g(1)$ $= 7$ c) $g(3)$ $= 1695$ d) $g(-1)$ $= 7$

$$\begin{array}{r|rrrrrrr} 2 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 4 & 8 & 22 & 44 & 86 & 172 \\ \hline & 2 & 4 & 11 & 22 & 43 & 86 & 175 \end{array}$$

(4) Find **ALL** real zeros of the polynomial function.

$f(x) = 4x^3 + 7x^2 - 11x - 18$

$$\frac{P}{Q} = \frac{\pm 1, 2, 3, 6, 9, 18}{\pm 1, 2, 4}$$

$$\begin{array}{r|rrrr} \boxed{-2} & 4 & 7 & -11 & -18 \\ & & -8 & 2 & 18 \\ \hline & 4 & -1 & -9 & 0 \end{array}$$

$$4x^2 - x - 9 = 0$$

$$x = \frac{1 \pm \sqrt{1 + 144}}{8}$$

$$\boxed{= \frac{1 \pm \sqrt{145}}{8}}$$