

The Rational Root Theorem

State the possible rational zeros for each function.

1) $f(x) = 3x^2 + 2x - 1$

2) $f(x) = x^6 - 64$

3) $f(x) = x^2 + 8x + 10$

4) $f(x) = 5x^3 - 2x^2 + 20x - 8$

5) $f(x) = 4x^5 - 2x^4 + 30x^3 - 15x^2 + 50x - 25$

6) $f(x) = 5x^4 + 32x^2 - 21$

7) $f(x) = x^3 - 27$

8) $f(x) = 2x^4 - 9x^2 + 7$

State the possible rational zeros for each function. Then find all rational zeros.

9) $f(x) = x^3 + x^2 - 5x + 3$

10) $f(x) = x^3 - 13x^2 + 23x - 11$

11) $f(x) = x^3 + 4x^2 + 5x + 2$

12) $f(x) = 5x^3 + 29x^2 + 19x - 5$

13) $f(x) = 4x^3 - 9x^2 + 6x - 1$

14) $f(x) = 3x^3 + 11x^2 + 5x - 3$

15) $f(x) = 5x^4 - 46x^3 + 84x^2 - 50x + 7$

16) $f(x) = 3x^4 - 10x^3 - 24x^2 - 6x + 5$

17) $f(x) = 3x^3 + 9x^2 + 4x + 12$

18) $f(x) = 2x^3 + 9x^2 + 19x + 15$

Critical thinking question:

- 19) In the process of solving $2x^3 + 7x^2 + 9x + 10 = 0$ you test 1, 2, 5, and 10 as possible zeros and determine that none of them are actual zeros. You then discover that $-\frac{5}{2}$ is a zero. You calculate the depressed polynomial to be $2x^3 + 2x + 4$. Do you need to test 1, 2, 5, and 10 again? Why or why not?

The Rational Root Theorem

State the possible rational zeros for each function.

1) $f(x) = 3x^2 + 2x - 1$

$$\pm 1, \pm \frac{1}{3}$$

2) $f(x) = x^6 - 64$

$$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64$$

3) $f(x) = x^2 + 8x + 10$

$$\pm 1, \pm 2, \pm 5, \pm 10$$

4) $f(x) = 5x^3 - 2x^2 + 20x - 8$

$$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{4}{5}, \pm \frac{8}{5}$$

5) $f(x) = 4x^5 - 2x^4 + 30x^3 - 15x^2 + 50x - 25$

$$\pm 1, \pm 5, \pm 25, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{25}{2}, \pm \frac{1}{4}, \pm \frac{5}{4}, \pm \frac{25}{4}$$

6) $f(x) = 5x^4 + 32x^2 - 21$

$$\pm 1, \pm 3, \pm 7, \pm 21, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{7}{5}, \pm \frac{21}{5}$$

7) $f(x) = x^3 - 27$

$$\pm 1, \pm 3, \pm 9, \pm 27$$

8) $f(x) = 2x^4 - 9x^2 + 7$

$$\pm 1, \pm 7, \pm \frac{1}{2}, \pm \frac{7}{2}$$

State the possible rational zeros for each function. Then find all rational zeros.

9) $f(x) = x^3 + x^2 - 5x + 3$

Possible rational zeros: $\pm 1, \pm 3$
Rational zeros: $\{-3, 1 \text{ mult. } 2\}$

10) $f(x) = x^3 - 13x^2 + 23x - 11$

Possible rational zeros: $\pm 1, \pm 11$
Rational zeros: $\{1 \text{ mult. } 2, 11\}$

11) $f(x) = x^3 + 4x^2 + 5x + 2$

Possible rational zeros: $\pm 1, \pm 2$ Rational zeros: $\{-1 \text{ mult. } 2, -2\}$

12) $f(x) = 5x^3 + 29x^2 + 19x - 5$

Possible rational zeros: $\pm 1, \pm 5, \pm \frac{1}{5}$ Rational zeros: $\left\{\frac{1}{5}, -5, -1\right\}$

13) $f(x) = 4x^3 - 9x^2 + 6x - 1$

Possible rational zeros: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}$ Rational zeros: $\left\{1 \text{ mult. } 2, \frac{1}{4}\right\}$

14) $f(x) = 3x^3 + 11x^2 + 5x - 3$

Possible rational zeros: $\pm 1, \pm 3, \pm \frac{1}{3}$ Rational zeros: $\left\{\frac{1}{3}, -3, -1\right\}$

15) $f(x) = 5x^4 - 46x^3 + 84x^2 - 50x + 7$

Possible rational zeros: $\pm 1, \pm 7, \pm \frac{1}{5}, \pm \frac{7}{5}$ Rational zeros: $\left\{\frac{1}{5}, 7, 1 \text{ mult. } 2\right\}$

16) $f(x) = 3x^4 - 10x^3 - 24x^2 - 6x + 5$

Possible rational zeros: $\pm 1, \pm 5, \pm \frac{1}{3}, \pm \frac{5}{3}$ Rational zeros: $\left\{\frac{1}{3}, 5, -1 \text{ mult. } 2\right\}$

17) $f(x) = 3x^3 + 9x^2 + 4x + 12$

Possible rational zeros:

 $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$ Rational zeros: $\{-3\}$

18) $f(x) = 2x^3 + 9x^2 + 19x + 15$

Possible rational zeros:

 $\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$ Rational zeros: $\left\{-\frac{3}{2}\right\}$ **Critical thinking question:**

19) In the process of solving $2x^3 + 7x^2 + 9x + 10 = 0$ you test 1, 2, 5, and 10 as possible zeros and determine that none of them are actual zeros. You then discover that $-\frac{5}{2}$ is a zero. You calculate the depressed polynomial to be $2x^3 + 2x + 4$. Do you need to test 1, 2, 5, and 10 again? Why or why not?

No. That would be like factoring 740 and discovering 3 isn't a factor but then checking if anything 740 breaks down into has a factor of 3. If the original problem doesn't have a factor of 3 then nothing it factors into will have a factor of 3.