Section 2.2 Polynomial Functions of Higher Degree

Objective: In this lesson you learned how to sketch and analyze graphs of polynomial functions

Important Vocabulary			
Continuous	**Extrema**	**Relative minimum**	**Relative maximum**
Repeated zero	**Multiplicity**	**Intermediate Value Theorem**	

I. Graphs of Polynomial Functions

What you should learn:

How to use transformations to sketch graphs of polynomial functions

Name two basic features of the graphs of polynomial functions.

1) Continuous

2) SMOOTH, ROUNDED CURVES.

Will the graph of $g(x) = x^7$ look more like the graph of $f(x) = x^2$ or the graph of $f(x) = x^3$? Explain.

$f(x) = x^3$ because both exponents are odd

II. The Leading Coefficient Test

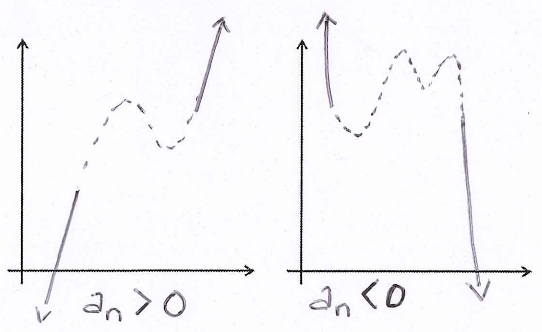
What you should learn:

How to use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions

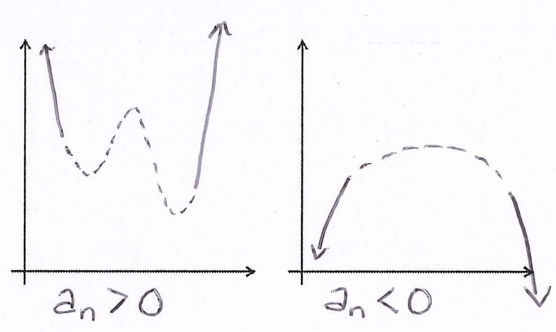
State the **Leading Coefficient Test:**

AS X MOVES WITHOUT BOUND TO THE LEFT OR RIGHT, THE GRAPH OF THE POLYNOMIAL FUNCTION $f(x) = a_n x^n + \cdots + a_1 x + a_0$, $a_n \neq 0$ EVENTUALLY RISES OR FALLS IN THE FOLLOWING MANNER.

1) When n is odd



2) When n is even



III. Zeros of Polynomials Functions

What you should learn:

How to find and use zeros of polynomial functions as sketching aids

Let f be a polynomial function of degree n. The function f has at most ___N___ real zeros. The graph of f has at most ___N-1___ relative **extrema**.

Let f be a polynomial function and let a be a real number. List four equivalent statements about the real zeros of f.

ALL MEAN THE SAME THING.

1) $x=a$ IS A ZERO of the FUNCTION f.

2) $x=a$ is a SOLUTION OF THE POLYNOMIAL EQUATION $f(x)=0$

3) $(x-a)$ is a factor OF THE POLYNOMIAL $f(x)$

4) $(a,0)$ is an X-INTERCEPT OF THE graph of f.

For a polynomial function, a factor of $(x-a)^k, k>1$, yields a **repeated zero**:

$x=a$ of MULTIPLICITY K

1) If K is $\boxed{\text{ODD,}}$ THEN THE GRAPH CROSSES THE X-AXIS AT $x=a$.

2) If K is $\boxed{\text{EVEN,}}$ THEN THE GRAPH TOUCHES THE X-AXIS AT $x=a$.

If a polynomial function f has a repeated zero $x=3$ with multiplicity 4, the graph of f ___TOUCHES___ the x-axis at $x=$ ___3___. If f has a repeated zero $x=4$ with multiplicity 3, the graph of f ___CROSSES___ the x-axis at $x=$ ___4___.

Basics:
Polynomial functions = continuous
So, allows you to draw lines between 2 points

IV. The Intermediate Value Theorem

What you should learn:

How to use the Intermediate Value Theorem to help locate zeros of polynomial functions

State the Intermediate Value Theorem: Let a and b be real numbers such that $a<b$. If f is a polynomial function such that $f(a) \neq f(b)$, then in the interval $[a,b]$, f takes on every value between $f(a)$ and $f(b)$.

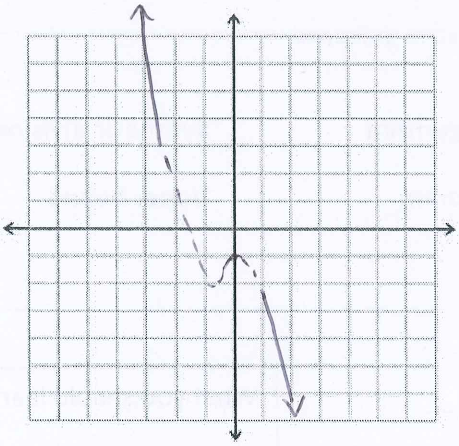
Describe how the Intermediate Value Theorem can help in locating the real zeros of a polynomial function f.

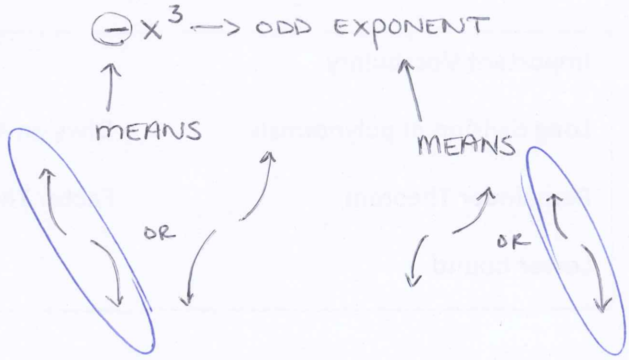
If you can find a value $x=a$, where a polynomial function is positive and a value $x=b$, where a polynomial function is negative, then you can conclude there exists a real zero between a and b.

Section 2.2 Examples – Polynomial Functions of Higher Degree

(1) Use the Leading Coefficient Test to determine the right and left hand behavior of the graph. Then sketch what the rest of the graph might look like.

$f(x) = -x^3 + 2x - 1$





(2) Find all the real zeros of the polynomial function.

$h(t) = t^2 - 6t + 9$

$t^2 - 6t + 9 = 0$

$(t-3)^2 = 0$

$t - 3 = 0$

$\boxed{t = 3}$

(3) Find a polynomial function with the given zeros, multiplicities, and degree.

Zero: -1, multiplicity: 2

Zero: -2, multiplicity: 1

Degree: 3

$f(x) = (x+1)^2(x+2)$

$= (x^2+2x+1)(x+2)$

$= x^3 + 2x^2 + 2x^2 + 4x + x + 2$

$\boxed{= x^3 + 4x^2 + 5x + 2}$