Section 1.6 Inverse Functions

Objective: In this lesson you learned how to find inverse functions graphically and algebraically

Important Vocabulary		
Inverse Function	**One-to-one**	**Horizontal Line Test**

I. Inverse Function

What you should learn:

How to find inverse functions informally and verify that two functions are inverse functions of each other

For a function f that is defined by a set of ordered pairs, to form the inverse function of f:

INTERCHANGE THE x AND y-VALUES FOR EACH ORDERED PAIR

For a function f and its inverse f^{-1}, the domain of f is equal to the range of f^{-1}, and the range of f is equal to the domain of f^{-1}.

To verify that two functions, f and g, are inverse functions of each other:

find $f(g(x))$ and $g(f(x))$. If both equal x then f and g are inverses

II. The Graph of an Inverse Function

What you should learn:

How to use graphs of functions to decide whether functions have inverse functions

If the point (a, b) lies on the graph of f, then the point (B , a) lies on the graph of f^{-1} and vice versa. The graph of f^{-1} is a reflection of the graph of f in the line $y=x$.

III. The Existence of an Inverse Function

What you should learn:

How to determine if functions are one-to-one

If a function is **one-to-one**, that means:

NO TWO ELEMENTS IN THE DOMAIN CORRESPOND TO THE SAME ELEMENT IN THE RANGE

FANCY TALK FOR, "EACH INPUT HAS A UNIQUE OUTPUT" i.e. x^2

A function f has an inverse f^{-1} if and only if:

it is one-to-one

To tell whether a function is one-to-one from its graph:

COMPLETE THE HORIZONTAL LINE TEST

IV. Finding Inverse Functions Algebraically

What you should learn:

How to find inverse functions algebraically

To find the inverse of a function f algebraically:

1) USE THE HORIZONTAL LINE TEST
2) REPLACE $f(x)$ WITH y
3) INTERCHANGE (switch) x AND y.
4) Solve for y. (get $y=$)
5) REPLACE y with $f^{-1}(x)$ AND VERIFY f AND f^{-1} ARE INVERSES

THIS NOTATION IS NOT OPTIONAL

Section 1.6 Examples – Inverse Functions

(1) Show that f and g are inverse functions (a) algebraically and (b) numerically.

$$f(x) = \frac{x-9}{4} \qquad g(x) = 4x + 9$$

a) $f(g(x)) = \frac{(4x+9)-9}{4}$

$= \frac{4x}{4} = x$ ✓

$g(f(x)) = 4\left[\frac{x-9}{4}\right]+9$

$= x-9+9$

$= x$ ✓

$\boxed{\text{INVERSES}}$

B) $f(5) = \frac{5-9}{4} = -1$

$g(-1) = 4(-1) = 5$ ✓

$g(5) = 4(5)+9 = 29$

$f(29) = \frac{29-9}{4} = 5$ ✓

$\boxed{\text{INVERSES}}$

(2) Determine whether the given function is one-to-one. If so, then find the functions inverse.

$$f(x) = 3x + 5$$

FOR ONE-TO-ONE WANT TO SHOW $f(a) = f(B)$ LEADS TO $a = B$

$3a + 5 = 3B + 5$ ← SOLVE LIKE ANY "NORMAL" EQUATION

$-5 \qquad -5$

$\frac{3a}{3} = \frac{3B}{3}$

$a = B$ ✓ ONE-TO-ONE

$y = 3x + 5$

$x = 3y + 5$

$3y = x - 5$

$y = \frac{x-5}{3}$

$$\boxed{f^{-1}(x) = \frac{x-5}{3}}$$