Section 1.5 Combinations of Functions

Objective: In this lesson you learned how to find arithmetic combinations and compositions of functions

I. Arithmetic Combinations of Functions

What you should learn:

How to add, subtract, multiply and divide functions

Just as two real numbers can be combined by the operations of additions, subtraction, multiplication, and division to form other real numbers, two functions f and g can be combined to create new functions such as the SUM, DIFFERENCE, PRODUCT, & QUOTIENT of f and g to create new functions.

The domain of an arithmetic combination of functions f and g consists of:

ALL REAL NUMBERS THAT ARE COMMON TO THE DOMAINS of f AND g.

Let f and g be two functions with overlapping domains. Complete the following arithmetic combinations of f and g for all x common to both domains:

1) Sum: $(f+g)(x) =$ $f(x)+g(x)$

2) Difference: $(f-g)(x) =$ $f(x)-g(x)$

3) Product: $(fg)(x) =$ $f(x)\cdot g(x)$

4) Quotient: $\left(\frac{f}{g}\right)(x) =$ $\frac{f(x)}{g(x)}, g(x)\neq 0$ ← MUST INCLUDE!

II. Compositions of Functions

What you should learn:

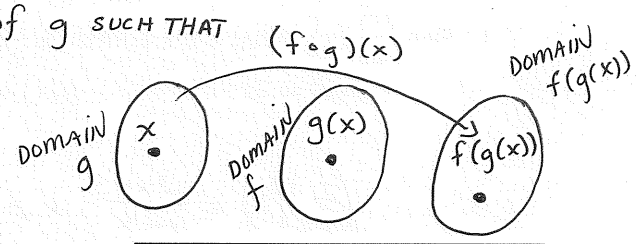
How to find compositions of one functions with another function

The **composition** of the function f with the function g is

$(f \circ g)(x) =$ $f(g(x))$

For the composition of the function f with g, the domain of $f \circ g$ is:

THE SET OF ALL x IN THE DOMAIN of g SUCH THAT $g(x)$ IS IN THE DOMAIN of f.



III. Applications of Combinations of Functions

What you should learn:

How to use combinations of functions to model and solve real-life problems

The function $f(x) = 0.06x$ represents the sales tax owed on a purchase with a price tag of x dollars and the function $g(x) = 0.75x$ represents the sale price of an item with a price tag of x dollars during a 25% off sale. Using one of the combinations of functions discussed in this section, write the function that represents the sales tax owed on an item with a price tag of x dollars during a 25% off sale.

$$f(g(x)) = 0.06(0.75x)$$

Section 1.5 Examples – Combinations of Functions

(1) Find $\left(\frac{f}{g}\right)(x)$. What is the domain of $\frac{f}{g}$?

$f(x) = 2x - 5$ $\quad$ $g(x) = 1 - x$

$\left(\frac{f}{g}\right)(x) = \frac{2x-5}{1-x}, \; x \neq 1$

SET BUILDER NOTATION: $\{x \mid x \neq 1\}$

INTERVAL NOTATION : D: $(-\infty, 1) \cup (1, +\infty)$

(2) Determine the domains of (a) f, (b) g, and (c) $f \circ g$

$f(x) = \sqrt{x+3}$ $\quad$ $g(x) = \frac{x}{2}$

a) D: $[-3, +\infty)$

b) D: $(-\infty, +\infty)$

c) $f(g(x)) = \sqrt{\frac{x}{2} + 3}$

$\frac{x}{2} + 3 \geq 0$

$x \geq -6$

D: $[-6, +\infty)$