Section 1.3 Graphs of Functions

Objective: In this lesson you learned how to analyze the graphs of functions

Important Vocabulary		
Graph of a Function	**Greatest Integer Function**	**Step Function**
Even Function	**Odd Function**	**Interval Notation**

I. The Graph of a Function

What you should learn:

How to find the domains and ranges of functions and how to use the Vertical Line Test for functions

Explain the use of open or closed dots in the graphs of functions:

- INDICATES THE GRAPH DOES NOT EXTEND BEYOND THOSE POINTS
- ALSO INDICATES VALUES THAT MAY OR MAY NOT BE INCLUDED IN THE DOMAIN & RANGE.

To find the domain of a function from its graph:

FIND WHERE THE X-COORDINATES BEGIN (LEFT SIDE OF GRAPH) AND FOLLOW THEM (TOWARDS THE RIGHT) TO THE END OF THE GRAPH.

To find the range of a function from its graph:

FIND THE LOWEST POINT ON THE GRAPH AND FOLLOW THE GRAPH TO THE HIGHEST POINT.

The **Vertical Line Test** for functions states:

A SET OF POINTS IN A COORDINATE PLANE IS THE GRAPH OF Y AS A FUNCTION OF X IF AND ONLY IF NO VERTICAL LINE INTERSECTS THE GRAPH AT MORE THAN ONE POINT

II. Increasing and Decreasing Functions

ALWAYS go LEFT TO RIGHT

What you should learn:

How to determine intervals on which functions are increasing, decreasing, or constant

A function f is **increasing** on an interval if, for any x_1 and x_2 in the interval:

$$x_1 < x_2 \text{ implies } f(x_1) < f(x_2)$$

"THE GRAPH IS GOING IN AN UPWARD DIRECTION"

A function f is **decreasing** on an interval if, for any x_1 and x_2 in the interval:

$$x_1 < x_2 \text{ implies } f(x_1) > f(x_2)$$

"THE GRAPH IS GOING IN A DOWNWARD DIRECTION."

A function f is **constant** on an interval if, for any x_1 and x_2 in the interval:

$$f(x_1) = f(x_2)$$

"THE GRAPH IS GOING IN A HORIZONTAL DIRECTION"

Given a graph of a function, how do you determine when a function is *increasing, decreasing*, or *constant*?

START ON THE LEFT HAND SIDE of the graph AND FOLLOW it towards the Right. If the graph is going down (decreasing); going up (increasing); going horizontal (CONSTANT)

III. Relative Minimum and Maximum Values

What you should learn:

How to determine relative minimum and relative maximum values of functions

A function value $f(a)$ is called a **relative minimum** of f if:

THERE EXISTS AN INTERVAL (x_1, x_2)

THAT CONTAINS a SUCH THAT

$x_1 < x < x_2$ IMPLIES $f(a) \leq f(x)$

INTERVAL NOTATION

A function value $f(a)$ is called a **relative maximum** of f if:

THERE EXISTS AN INTERVAL (x_1, x_2) THAT CONTAINS a

SUCH THAT

$x_1 < x < x_2$ IMPLIES $f(a) \geq f(x)$

The point at which a function changes from increasing to decreasing is a relative MAXIMUM. The point at which a function changes from decreasing to increasing is a relative MINIMUM.

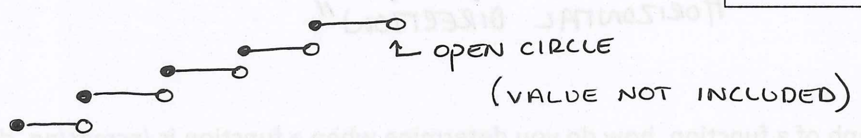
IV. Step Functions and Piecewise-Defined Functions

What you should learn:

How to identify and graph step functions and other piecewise-defined functions

Describe the graph of the greatest integer function:

LOOKS LIKE A SET OF STAIRS (STEPS)



Describe the graph of a piecewise-defined function:

A GRAPH OF EACH EQUATION (PIECE) USING THE APPROPRIATE DOMAIN ON THE SAME SET OF AXES

V. Even and Odd Functions

What you should learn:
How to identify even and odd functions

A graph is symmetric with respect to the y-axis if, whenever (x, y) is on the graph, the point $(-x, y)$ is also on the graph. A graph is symmetric with respect to the x-axis if, whenever (x, y) is on the graph, the point $(x, -y)$ is also on the graph. A graph is symmetric with respect to the origin if, whenever (x, y) is on the graph, the point $(-x, -y)$ is also on the graph. A graph that is symmetric with respect to the x-axis is:

NOT THE GRAPH OF A FUNCTION ($y=0$ is the EXCEPTION)

↳ WILL ALWAYS FAIL THE VERTICAL LINE TEST.

A function is **even** if, for each x in the domain of f, $f(-x) =$ $f(x)$

A function is **odd** if, for each x in the domain of f, $f(-x) =$ $-f(x)$

Section 1.3 Examples – Graphs of Functions

(1) Find the domain and range of the function, then graph. Write your domain and range in interval notation.

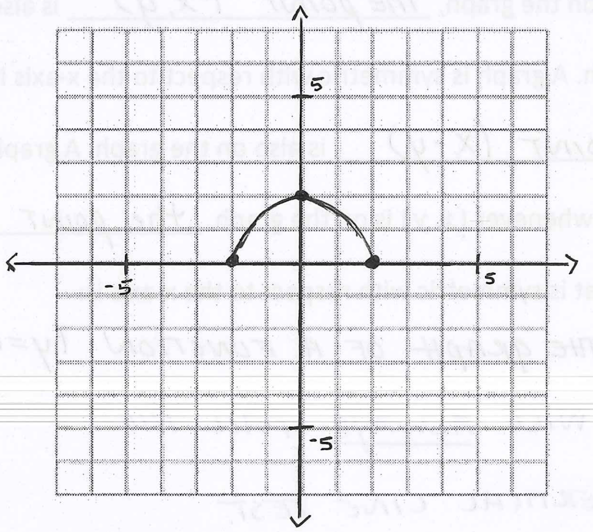
$$f(x) = \sqrt{4 - x^2}$$

$4 - x^2 = 0$

$-x^2 = -4$

$x^2 = 4$

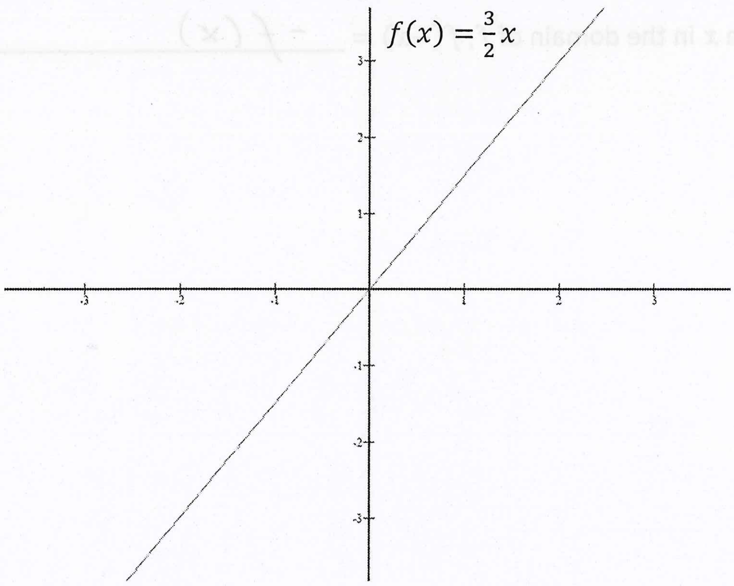
$x = \pm 2$



$D: [-2, 2]$

$R: [0, 2]$

(2)Determine the open interval(s) on which the function is *increasing*, *decreasing*, or *constant*.



INCREASING

$(-\infty, +\infty)$

$\mathbb{R}$ IN INTERVAL NOTATION

(3) Tell whether the function is *even*, *odd*, or *neither* algebraically.

$$f(t) = t^2 + 2t - 3$$

EVEN ✓

$f(-t) = (-t)^2 + 2(-t) - 3$

$= t^2 - 2t - 3$

NO

odd ✓

$f(-t) = t^2 - 2t - 3$

$= -[-t^2 + 2t + 3]$

NO

EVEN

$f(x) = f(-x)$

ODD

$f(x) = -f(x)$

$\boxed{\text{NEITHER}}$